Feature Article

Reflection Coefficient Analysis of the Effect of Ground on Antenna Patterns

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INTRODUCTION

Calculating the effect of a real (i.e., finite conductance) ground on antenna radiation patterns by means of reflection coefficients is simple and has been known for some time. However, there appear to be few computer programs which actually use the technique to calculate the far field radiation patterns of antennas above a real ground. This may be due to the fact that when the antenna is close to ground, the resulting accuracy is marginal when compared with the method of moments (usually NEC, Numerical Electromagnetics Code).

The advantage of using reflection coefficients rather than the method of moments is in the time required for analysis. To perform a reflection coefficient analysis, we simply add a direct ray with a reflected wave of the appropriate magnitude and phase. No matrix operations are required. In addition, because no matrices are involved, memory requirements are reduced allowing implementation on a small personal computer. The memory which is left can now be used for additional user interface, for example, menus, forms, graphics and disc access.

A program was written on an Apple II+ using reflection coefficients to calculate the far field radiation patterns of dipoles, monopoles (half of a dipole), isotropic antennas and arrays of these antennas. The entire program was written in assembly language (the reason for this choice is discussed later). Subsequently, the program was transported to the Commodore-64 and the IBM-PC.

The program was described in a series of QST articles [1]-[5] and was offered commercially [6] to (primarily) the amateur radio community. The intent was to stimulate interest in the mathematical aspects of antenna design, especially in the younger, pre-college readers. The program was reviewed in several other amateur radio magazines [7],[8] and over 200 copies have been sold world wide. The series of articles generated over 1000 pieces of correspondence.

While intended for the use of individual radio amateurs, about a third of the users are commercial in nature ranging from small consulting companies to large defense firms. In addition, several community colleges and universities have adopted the program in their curriculum.

The name selected for the program is Annie. A proper name was selected because the author prefers, when possible, to refrain from the use of acronyms. Note that as a proper name, only the first character is capitalized.

This paper will describe the theory used by Annie and then provide some examples.

THEORY

The analysis of antenna patterns using reflection coefficients is described in Jordan and Balmain [9], pp. 630-644. Summarizing, the total far field in a given direction consists of a direct ray and a reflected ray. The reflected ray is modified in phase by the additional distance that the ray must travel before being reflected. Upon reflection, the ray is modified in both phase and magnitude by the reflection coefficient of the ground at the point of reflection. The reflection coefficient depends on the conductivity and dielectric constant of the ground, the angle of incidence and the polarization of the incident ray.

The equations for the reflection coefficients for horizontally (R_H) and vertically (R_V) polarizied rays are

Introducting James C. Rautio
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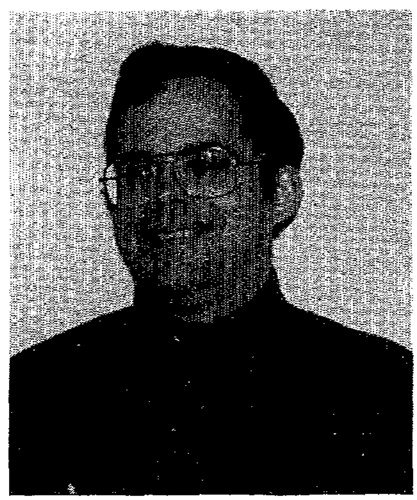


James C. Rautio received a BSEE from Cornell University in 1978, an MS in Systems Engineering from University of Pennsylvania in 1981 and a Ph.D. in Electrical Engineering at Syracuse University in 1986. His dissertation topic was "A Time-harmonic Electromagnetic Analysis of Shielded Microstrip Circuits". In 1978 he joined General Electric Space Division where he worked on filters, low noise amplifiers, millimeter wave automated network analyzers and microwave design software. In 1982, he joined General Electric Electronics Laboratory where he designed GaAs monolithic microwave integrated circuits and developed microwave software. In 1986 he became an assistant professor at Syracuse University. He has been licensed as a radio amateur, AJ3K, since 1969.

DEFINE ARRAY

ARRAY>A ELEMENT 1

DIPOLE .50000 WAVELENGTHS LONG

ELEMENT POSITION ELEMENT ROTATION

X = 0.0 PHI = 0.0

Y = 0.0 THETA = 90.000

Z = 0.0

ELEMENT WEIGHT = 1.0000 AT 0.0 DEG.

=

Figure 1. Annie's Define Array menu (or form) allows the user to view the value of all important parameters and to change those of interest.

$$R_H = \frac{\sin\psi - \sqrt{(\epsilon_r - jY) - \cos^2\psi}}{\sin\psi + \sqrt{(\epsilon_r - jY) - \cos^2\psi}}$$

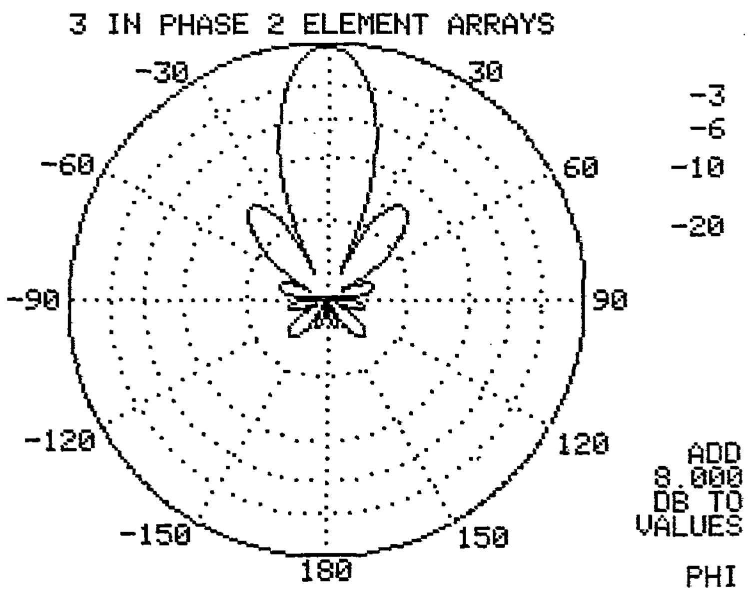
$$R_V = \frac{(\epsilon_r - jY)\sin\psi - \sqrt{(\epsilon_r - jY) - \cos^2\psi}}{(\epsilon_r - jY)\sin\psi + \sqrt{(\epsilon_r - jY) - \cos^2\psi}}$$

where ψ = angle of the reflected wave, measured from horizontal
ϵ_r = relative dielectric constant of the ground
$Y = 18\ X$
$X = \sigma/f$
σ = conductivity of the ground in mS/m
f = frequency in MHz.

The above reflection coefficients are complex quantities. The reflected wave should be multiplied by the reflection coefficient and then the phase corresponding to the additional distance traveled by the reflected wave should be added to the phase of the result. The additional phase is

$$\Phi = 2\beta\left[\left(x\cos(\phi) + y\sin(\phi)\right)\sin(\theta) + z\cos(\theta)\right]$$

where x,y,z = coordinates of antenna center
$\beta = 2\pi/\lambda$
ϕ,θ = azimuth and zenith angle of desired far field direction.



**** ARRAY A ****

1) DIPOLE, LENGTH=.50000, X= 0.0, Y= 0.0, Z=1.0000, PHI=90.000, THETA=90.000, WEIGHT=1.0000, ANGLE= 0.0

2) DIPOLE, LENGTH=.50000, X=-.2500, Y= 0.0, Z=1.0000, PHI=90.000, THETA=90.000, WEIGHT=1.0000, ANGLE=90.000

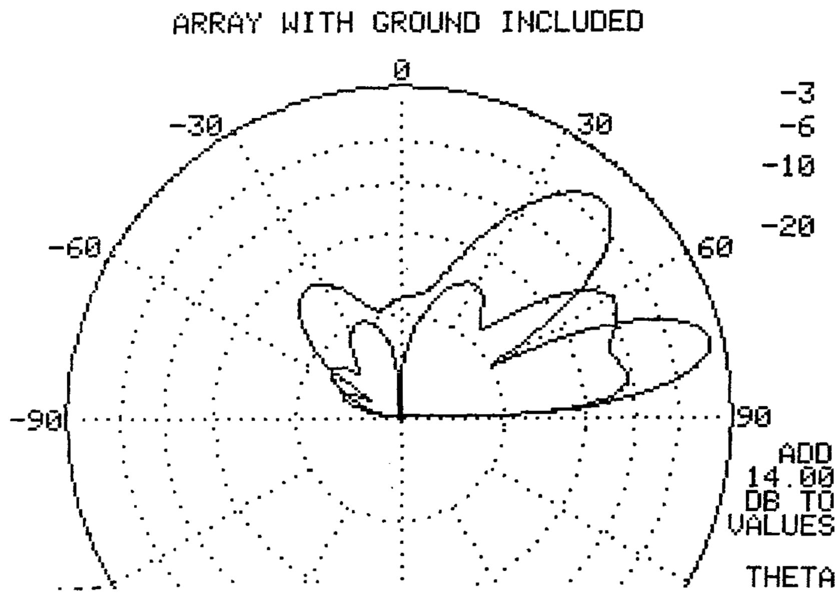
**** ARRAY B ****

1) ARRAY A, LENGTH=.50000, X= 0.0, Y= 0.0, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

2) ARRAY A, LENGTH=.50000, X= 0.0, Y=.75000, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

3) ARRAY A, LENGTH=.50000, X= 0.0, Y=-.7500, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

Figure 2. This six element array in free space is an example of Annie's plotting capability.



**** ARRAY A ****

1) DIPOLE, LENGTH=.50000, X= 0.0, Y= 0.0, Z=1.0000, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

2) DIPOLE, LENGTH=.50000, X=-.2500, Y= 0.0, Z=1.0000, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE=90.000

**** ARRAY B ****

1) ARRAY A, LENGTH=.50000, X= 0.0, Y= 0.0, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

2) ARRAY A, LENGTH=.50000, X= 0.0, Y=.75000, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

3) ARRAY A, LENGTH=.50000, X= 0.0, Y=-.7500, Z= 0.0, PHI= 0.0, THETA= 0.0, WEIGHT=1.0000, ANGLE= 0.0

Figure 3. A vertical (theta) cut of the six element array of Figure 2 when average ground is added. The large lobed pattern uses horizontal dipole elements while the dipoles are vertical for the other pattern.

When the direct and reflected wave are added together, we have the far field strength in the given direction. To calculate the magnitude of the direct wave (and of the reflected wave, prior to reflection), we use the usual equation [9], pg. 355, for a vertically oriented dipole far field pattern

$$P = \frac{\cos(\beta H \cos\theta) - \cos\beta H}{\sin\theta}$$

where P = Pattern value, a quantity proportional to the far field
H = half the dipole length

For $\sin\theta = 0.0$, the above expression becomes indeterminant. In such a case, the following approximation provides an accuracy of ±.005 dB for angles which are within 5 degrees of 0.0

$$P \cong \beta H\theta \Big(\sin(\beta H(\cos\theta + 1)/2) \Big) / 2$$

If we consider only half of a dipole (a monopole) still with the same sinusoidal current distribution, the pattern becomes a complex quantity

$$P = \frac{\cos(\beta H \cos\theta) - \cos(\beta H)}{\sin\theta} + j\,\frac{\sin(\beta H \cos\theta) - \sin(\beta H)\cos\theta}{\sin\theta}$$

There are also small angle approximations for the above expressions. Arrays of half dipoles, properly oriented and phased are useful in the analysis of the inverted V and square loop antennas.

In general, we will want to analyze antennas which have been rotated from the vertical ($\theta = 0$) position for which the above equations are valid. Thus, we want to determine the angle, θ', to use in the above equations when we wish to determine the pattern value in the direction (ϕ,θ) given that the axis of the dipole has been rotated by zenith angle θ_R and azimuth angle ϕ_R. The transformation of θ effected by this rotation is given by [10], Appendix 5

$$\cos(\theta') = \cos(\theta)\cos(\theta_R) + \sin(\theta)\sin(\theta_R)\cos(\phi-\phi_R)$$

Since the reflection coefficients depend on the polarization, we must also determine how much of the incident wave is vertically and how much is horizontally polarized. This is determined as follows

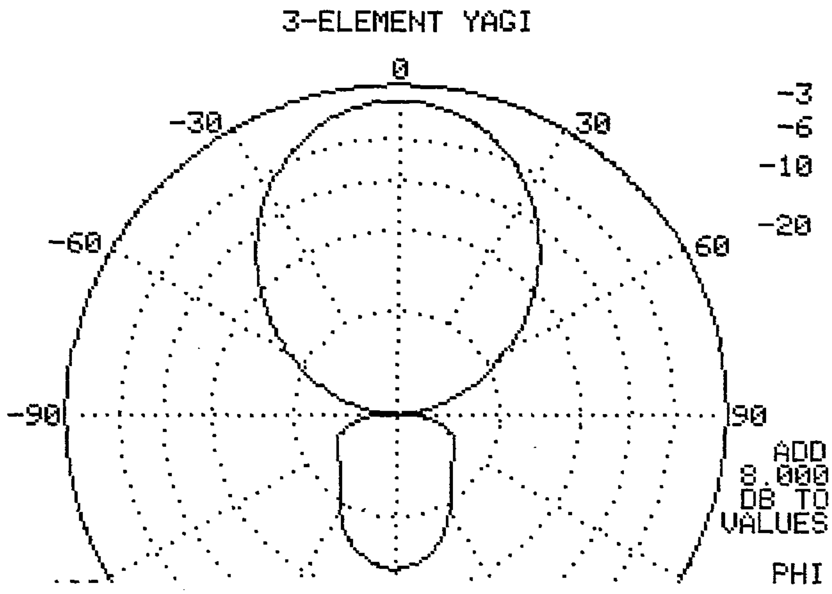
$$P_V = PA/Mag \qquad P_H = PB/Mag$$

$$A = -\sin(\theta_R)\sin(\phi-\phi_R)$$

$$B = \cos(\theta_R)\sin(\theta) - \sin(\theta_R)\cos(\theta)\cos(\phi-\phi_R)$$

$$Mag = \sqrt{A^2+B^2}$$

The above equation is closely related to equation 5A-9 of [10]. Care must be taken in the evaluation of the above equation in case Mag is close to zero.



**** ARRAY A ****

1) DIPOLE, LENGTH=.52310, X=-.2000, Y= 0.0, Z= 0.0, PHI=90.000, THETA=90.000, WEIGHT=2.3593, ANGLE=108.00

2) DIPOLE, LENGTH=.50220, X= 0.0, Y= 0.0, Z= 0.0, PHI=90.000, THETA=90.000, WEIGHT=12.000, ANGLE=313.00

3) DIPOLE, LENGTH=.48260, X=.20000, Y= 0.0, Z= 0.0, PHI=90.000, THETA=90.000, WEIGHT=6.1973, ANGLE=156.00

Figure 4. A Yagi antenna was analyzed by taking the phasings and weights for the elements from a MININEC calculation.

Thus an antenna pattern may now be calculated for a dipole with an arbitrary orientation by first transforming the direction of interest to θ' and calculating the value of the direct ray at the transformed angle. Then perform a similar calculation for the reflected ray, including the reflection coefficient and the extra distance traveled by the reflected ray. This must be done twice, once for each polarization. Then add the direct ray to the reflected ray and the pattern value in the given direction is evaluated.

The total antenna pattern, as well as just the vertical and horizontal components can now be quickly calculated. Two more quantities of interest are polarization aspect ratio and tilt. The aspect ratio is the ratio of the major to minor axes of the polarization ellipse while tilt is the angle the major axis of the ellipse makes with the horizontal (ϕ) direction. The equations for these quantities are given in [11], pp. 427-441

$$\text{Aspect Ratio} = \left(1 + \sqrt{1-C^2} \right)/C$$

$$\text{Tilt} = 0.5 \tan^{-1}\left[2P_V P_H \cos(\delta)/(P_H^2-P_V^2) \right]$$

$$\text{where } C = 2P_V P_H \sin(\delta)/(P_V^2+P_H^2)$$

δ = Phase by which vertical leads horizontal

For correct values of tilt, the inverse tangent should be evaluated using the FORTRAN ATAN2 function or equivalent.

IMPLEMENTATION

The above equations were implemented in assembly language on an Apple][+. Assembly language was chosen for its speed and compactness. The challenge of writing such a program in assembly language also played an important role.

To realize a fast analysis, a floating point package was written. This included specifying a floating point format and writing routines for the basic four arithmetic functions. Once those had been debugged and verified, the transcendental functions were implemented using closed form polynomial approximations [12]. Dealing directly with assembly language allowed the generation of highly efficient code.

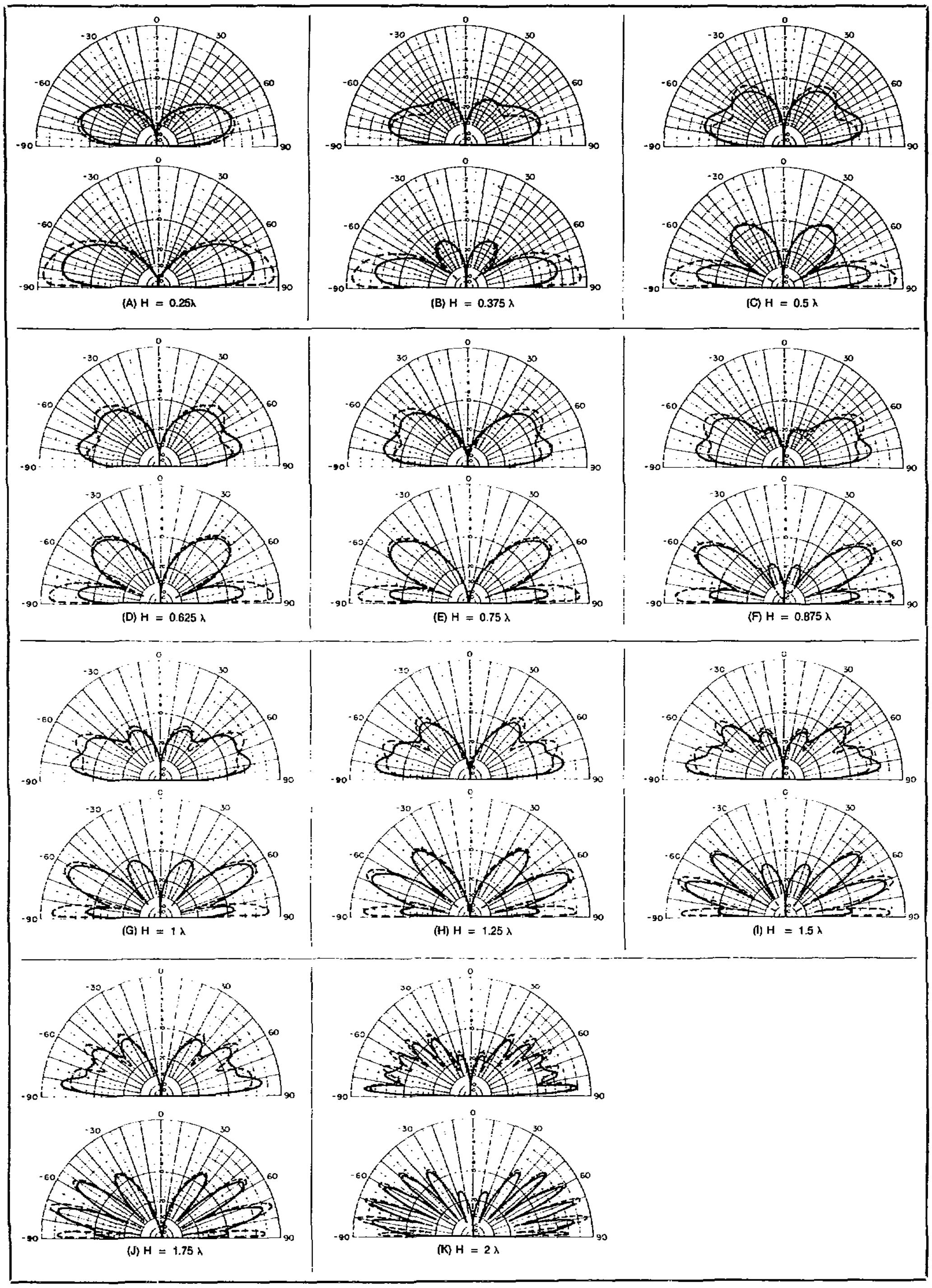
The compactness was required because we wished to include a substantial user interface. The program uses the entire 64K memory of the Apple, two thirds of which is devoted to the user interface. In addition, the user interface required fast screen manipulation, possible only with assembly language.

Fast screen manipulation is required to update forms. Unlike the usual question and answer session, a form displays all the critical parameters on the same screen. The user can then move a cursor about the screen at will, changing values to those desired. See Figure 1. Annie uses about a dozen forms to handle all the options.

Porting the software to the Commodore-64 (which uses the same microprocessor) was simple. A few I/O addresses were changed, the program was then assembled on the Apple and an image of the Apple's memory was copied directly to the C-64 using some custom hardware and software.

A significant disadvantage of assembly language is lack of portability when going to a machine with a different microprocessor. Such is the case with the IBM-PC. Thus, rather than staying with assembly language on the IBM-PC, we rewrote the program in Pascal (which tends to be more portable than FORTRAN, especially with regard to I/O, and also allows a more structured and supportable source than does FORTRAN).

A second disadvantage of assembly language is the



Figures 5 and 6 are from an article by the author in the August 1986 QST Magazine; used with permission.

Figure 5. Half-wavelength vertical dipoles. The height of the dipole center is shown at the bottom of each frame. Patterns for four grounds are shown. In the upper pattern of each frame, the solid line is poor ground (X=0.1, ϵ_R = 7) and the dashed line is good ground (X=1.0, ϵ_R = 15). In the lower pattern, the solid line is for very good ground (X=10.0, ϵ_R=30) and the dashed line is for sea water (X=1000, ϵ_R=80). Add 6 dB to the values shown.

time required to write software. The initial Apple version required seven months of intensive (after hours) work. The IBM version was written in two weeks (with the benifit of having already worked out the theory and user interface).

SOME EXAMPLES

An Annie analysis can be run with or without the effect of ground included. Figure 2 shows a six element array directivity (phi cut) pattern in free

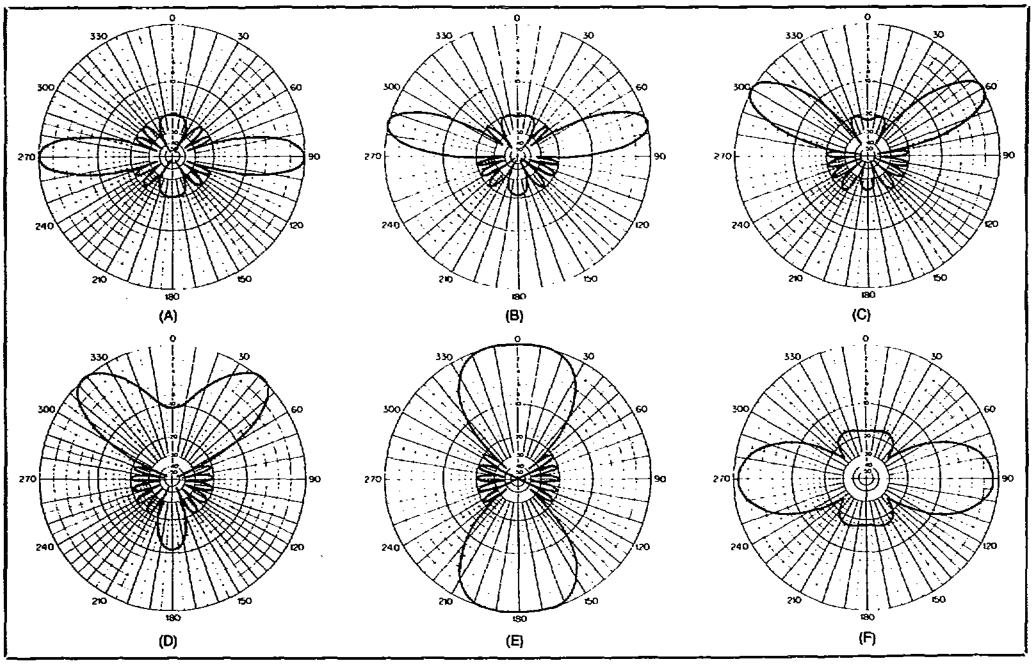


Figure 6. Chebyshev array in free space as the element phasings are changed. The elements are spaced at half wavelength intervals. At A, all the elements are in phase. Each element is given a progressive 45 degree phase shift in B. The shift is 90 degrees for C, 135 degrees for D and 180 degrees for F. At F, a poor ground causes the sidelobes to melt together.

space. The numbers along the right hand side are the number of dB each dotted circle is down from the solid, outer circle. The comment in the bottom right hand corner indicates that the outer circle is set at 8 dBd. The graphics options permit the plot to be positioned at any point on the screen and to be drawn with any magnification or aspect ratio (so a circle will be a circle).

The array of Figure 2 is actually a three element array with each of the three elements consisting of a two element array. Thus, we see that an array may have other arrays as elements. In fact, arrays of arrays may be nested to four levels with each level containing up to 16 elements. Thus the largest antenna that Annie could possibly analyze would include over 65,000 elements.

Figure 3 shows the same array, now at a height of one wavelength above average ground. The pattern is a vertical (theta cut) cross section. The larger pattern is with horizontal dipoles, the smaller pattern is with vertical dipoles. The washed out null in the vertically polarized pattern at θ = 70 degrees appears to be at pseudo Brewster's angle.

Annie does not calculate input impedances or coupling between elements. However one can use another program, such as MININEC (MINI-Numerical Electromagnetics Code) [13] to evaluate relative element phasings and weights. Figure 4 shows a three element Yagi. The phasings and weights were evaluated with MININEC and transferred to Annie. Once in Annie, we may now quickly obtain graphics and evaluate the effect of different heights and orientations above a real ground.

A word of caution, since Annie does not evaluate input impedances, comparisions between different antennas should be done with care.

The next examples were taken from the **QST** series [1]-[5] and reprinted with the permission of the ARRL (American Radio Relay League).

Figure 5 shows a theta cut (vertical cross section) of a vertical dipole at various heights above various qualities of ground.

Figure 6 shows a seven element Chebyshev array over perfect ground. The amplitudes were taken from an example given by Steinberg [14], pp. 111-119. The phases were changed from plot to plot, steering the beam. The last plot shows that a real ground tends to melt the sidelobes together.

While the reflection coefficient technique loses validity for very low antennas and does not calculate input impedance or mutual coupling, we see that a number of useful results can be obtained allowing us to take advantage of Annie's fast analysis, graphics and extensive user interface.

CONCLUSION

We have presented the equations required to perform a reflection coefficient analysis of antenna patterns which include the effect of real ground. We have also described a program, Annie, which implements the technique as well as presenting a number of analyses performed using the program.

TRADEMARK ACKNOWLEDGMENT

IBM-PC is a trademark of IBM and Apple][+ is a trademark of Apple Computer.

REFERENCES

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[3] Rautio, "The Effect of Real Ground on Antennas - Part 3," **QST**, June 1984, pp. 30-35.

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[6] Annie is available from Sonnet Software, 4397 Luna Course, Liverpool, NY 13090. The C-64 version is $39.95 and the Apple and IBM versions are $49.95. An IBM version which requires the 8087 coprocessor is $170.00. A 56 page manual is included. Some printers are not supported.

[7] Thurber, "Antennas: A Look at Annie," **CQ**, March 1985, pp. 60-63.

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Introducing Joseph A. Kinzel Feature Article Author

Mr. Kinzel is currently manager of phased array technology at General Electric's Electronics Laboratory in Syracuse, New York. He has responsibility for the identification and development of applications for microwave and millimeter-wave monolithic technology, which includes providing GaAs phased array module leadership involving C-band solid-state radar, Wideband module technology, X-band transmit/receive modules, 20 GHz module technology, plus array and module technology for missile seeker and V-band space-based radar applications.

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1987 URSI General Assembly Travel Information

The XXIInd General Assembly of the International Radio Science Union (URSI) will be held in Tel Aviv, Israel, August 24 - September 2, 1987 (see page 18 of the October, 1986 Newsletter for an announcement). A blue "First Announcement" booklet, describing the General Assembly, the technical program, and providing registration and travel information has been distributed. Copies are available from the Secretariat, URSI General Assembly, POB 50006, Tel Aviv 61500, Israel, telephone 03-654571, Telex 341171 KENS IL.

Kenes Travel of Israel is the agency identified in this announcement for those wishing to use their services in connection with travel. Note, however, that their United States representative is no longer that given in the above First Announcement. Their U.S. representative is now the following:

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