An Ultra-High Precision Benchmark for Validation of Planar Electromagnetic Analyses

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***Abstract*— A stripline standard is applied to the validation of planar electromagnetic analysis. Since an exact theoretical expression is available for stripline, a benchmark can be specified to the accuracy to which the expression can be evaluated. Data for the benchmark accurate to about 10^{-8} is provided. A definition for an error metric appropriate for use with the benchmark is illustrated. A means of calculating a precise value of analysis error using the error metric is described. A first order numerical value for the residual analysis error can also be obtained from the calculated S-parameters by inspection. The benchmark can be applied to any planar electromagnetic analysis capable of analyzing stripline. Example results, illustrating absolute convergence of an analysis to 0.05%, are provided.**

I. INTRODUCTION

RECENTLY, electromagnetic analysis has become a mainstream microwave design tool. In fact, in sharp contrast with the situation five years ago, electromagnetics is now often considered a critical part of the microwave design cycle. However, in spite of this sudden growth, there has been no attempt to seriously quantify the error of such analyses. Accuracy is usually only subjectively discussed with the ubiquitous, "Good Agreement Between Measured And Calculated," (GABMAC). Quantitative treatment of error has been almost entirely absent. In order to treat error quantitatively, we need two things: 1) a metric with which to measure error, and 2) a precise benchmark to which we may apply the metric. Neither exists with the present state of validation technique. With the strong dependence of today's microwave design on electromagnetic software, both are needed as a vital supplement to the usual GABMAC validation.

It is to be emphasized that the benchmark we describe is intended as a simple, quantitative benchmark. It does not represent an exhaustive suite of benchmarks. Additional benchmarks taking into account circuit complexity, loss, dispersion, metal thickness, etc., are still needed and should be the subject of future research. Unfortunately, it is unlikely that such benchmarks can be developed with the same degree of precision as this benchmark. Therefore, we place our initial concentration on the stripline benchmark.

II. THE ERROR METRIC

Error is usually specified in percent. In words, the percent error is the difference between the correct value and the calculated value divided by the correct value and multiplied by 100. One might first be tempted to apply this definition directly to S-parameter data. This can be difficult. As just one example, suppose we are evaluating the 50 Ohm S-parameters of a perfect 50 Ohm line. The correct value of S_{11} magnitude is 0.0. To calculate the percent error, we must divide by zero. Other definitions based directly on S-parameters encounter similar difficulties.

We suggest using an error metric based on quantities derived from the calculated S-parameters. For the special case of a simple transmission line, these quantities can be the characteristic impedance and the velocity of propagation. Given the calculated S-parameters for a 2-port transmission line, we can uniquely and unambiguously determine the equivalent characteristic impedance and velocity of propagation as described in [1] and [2]. Alternatively, any of the usual circuit theory based microwave programs may be used to optimize the characteristic impedance and velocity of propagation of an ideal TEM line so that the resulting S-parameters are identical to the calculated S-parameters at any given frequency.

Once the equivalent characteristic impedance and propagation velocity are obtained, percent error is calculated by differencing with the correct value for each. For example, suppose we are analyzing a line which we know to be exactly 50 Ohms and 100 degrees long. Let's say the calculated S-parameters exactly match those of a 51 Ohm line that is 101 degrees long. This means that there is 2% error in characteristic impedance and 1% error in propagation velocity. The total error of the analysis is 3%. The two errors are simply added for worse case because we have no information on correlation of the errors.

Once we have a value for the error, the designer can now estimate the accuracy of results for more complex circuits. For example, if the error is 3%, then the designer can have confidence that the calculated S-parameters correspond to a circuit whose transmission lines have a characteristic impedance and propagation velocity within 3% of their correct value. The impact of this error on S-parameters is strongly dependent on the specific circuit. Error bounds on the S parameters can be determined using the usual circuit theory based programs.

Alternatively, we may also look at the equivalent lumped circuit. In the case of a transmission line, we have an L-C-L-C-··· network. For more complex circuits, a lumped model of complexity sufficient to model the structure over the desired frequency range can be used. The model is usually fitted to

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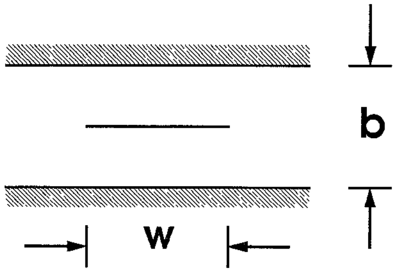

Fig. 1. Exact theoretical expressions are available for the stripline geometry.

the measured or calculated data by iterative techniques. More recently [3], closed form techniques can also be used. Once the equivalent lumped model is obtained, the total analysis error, to first order, is the maximum percent error of any single lumped element.

Given the value of the residual error of an analysis, an engineer can determine the accuracy of the calculated S-parameters, just as for the transmission line case. If an analysis has an error of 3%, the designer may conclude that the calculated S-parameters correspond to a lumped model all of whose elements are within 3% of their correct value. S-parameter error bounds must, again, be determined by application of circuit theory analysis.

To first order, both error definitions (lumped and distributed) are equivalent. If we were to apply this metric to a benchmark with loss (which is not the case here), the quantities of interest in both cases become complex and we specify the magnitude of the error.

III. The Stripline Benchmark

Use of the above error metric requires precise knowledge of the correct answer. This is difficult to achieve in an actual measurement. Thus, we use infinitely thin, lossless stripline, one of the few planar structures for which there is an exact theoretical solution [9]. The equation for stripline (Fig. 1) is

$$Z_0\sqrt{\epsilon_r} = \frac{\eta_0}{4}\frac{K(k')}{K(k)}$$

$$k = \tanh\left(\frac{\pi}{2}\frac{w}{b}\right) \qquad k' = \sqrt{1-k^2} \qquad \eta_0 = 376.7303136$$

$$K(k) = \text{Complete Elliptic Integral of the First Kind.}$$

(Note that the "30π" term frequently quoted in this expression is an approximation in error by 0.07%.) Using the above equations, obtaining the Elliptic Integral from [4] (note errata: $m_1 = 1 - m^2$, not $1 - m$), we obtain the results listed in Table I. The accuracy of the results is limited only by the accuracy of the numerical calculation, which, in this case, is about 10^{-8}. The accuracy of the expressions in [4] have been verified by direct summation of the Taylor series in quadruple precision for the cases cited in Table I.

In [13], results of application of the stripline benchmark are solicited for subsequent publication.

TABLE I
Stripline Width in Air for 1 mm Ground Plane Spacing

Z_0 (Ohms)	Width(mm)
25.00000	3.3260319
50.00000	1.4423896
100.0000	0.50396767

IV. Justification of Stripline as a Benchmark

By far, the most important desired characteristic of a benchmark is that the correct result be known with as much precision as possible. The ideal benchmark, in this regard, is one for which there is an exact answer. For scattering, there are many such problems. However, within the important class of 3-D planar structures, the only well known structure for which there is an exact solution is uniform, infinitely thin, lossless stripline.

A stripline standard provides the advantage of simplicity. This is important because once we have identified an error, we need to find the cause of that error. If the standard were complicated, there would be many confounding variables [12], making the task of attributing causality much more difficult. On the other hand, with such a simple circuit, we have the disadvantage of not including error sources due to the effect of circuit complexity, metal thickness, vertical (via) current, loss, and dispersion. Thus, it is important to keep in mind that analysis error in the representation of these effects is not evaluated. However, the benchmark does allow us to precisely quantify error due to subsection size (or other discretization) error, Green's function approximations, other approximations, side-wall coupling error (if present), de-embedding error, numerical precision error, and small magnitude programming errors. With the present generally accepted validation procedures, these error sources are rarely, if ever, quantitatively measured or even detected.

An advantage of being dispersion free is that there is no ambiguity as to the definition of characteristic impedance. In say, microstrip, there are at least four different definitions of characteristic impedance [10], [11], each of which gives different answers covering a range exceeding 20%. In spite of these difficulties, generating a microstrip standard to an accuracy of 10^{-4} is the subject of current research.

As for metal thickness, most planar analyses are for zero thickness conductors. Thus, the most useful benchmark would also be for zero thickness, as this stripline benchmark is. A non-zero thickness benchmark is, of course, also important and should be the subject of future research, however, it is secondary in importance to the zero thickness standard, and, again, we lack an exact theoretical solution.

As for the present state of validation, we often see only a comparison of calculated and measured data. Measurement error, which is rarely evaluated, can easily conceal any analysis error less than about 5%. While standard practice in the past, such a situation is now becoming unacceptable due to the application of such software in applications requiring 1% to

0.1% accuracy. Occasionally convergence analysis has been performed by decreasing subsection size and checking to see that the answer converges to a single answer, right or wrong. Absolute convergence analysis (checking that the analysis converges to the correct answer, not just any answer) is, to date, never done for planar problems.

For most published work this results in the ubiquitous, qualitative, "Good Agreement Between Measured And Calculated" (GABMAC). In contrast, the stripline standard allows precise, quantitative evaluation of analysis error due to a number of sources. Thus, engineering judgments as to "good" or "bad" can be made on the basis of hard data, not fuzzy feelings.

Since most existing electromagnetic analysis techniques have never been subjected to benchmarks capable of detecting error smaller than about 5%, we expect this benchmark, once applied, to result in the identification of more than a few unexpected error sources. One of the major error sources we expect to be found is de-embedding error. An approximate de-embedding can leave error in the 1% to 5% range, which is unlikely to be detected by the usual GABMAC validation. A coarse subsectioning can leave error in the 3% to 6% range. An example of the effect of such error is shown in the next to last section. A simple GABMAC validation, while still useful, is no longer sufficient for today's applications.

Incorrectly accounting for dispersion during de-embedding can leave errors of 5% or more. Unfortunately, this standard does not test the dispersive situation. An appropriate standard is the subject of current research.

V. Quick Application of the Stripline Benchmark

For the special case of a 50 Ohm stripline precisely a quarter wavelength long, a first order estimate of the analysis error may be read directly from the calculated S-parameters. In this case the magnitude of S_{11} represents the error in characteristic impedance and the difference of S_{21} phase from -90 degrees represents the propagation velocity error. For example, if S_{11} magnitude is 0.02, the impedance error is 2%. If S_{21} phase is -91 degrees, the velocity error is 1%. The total error is then 3%.

To quickly apply the stripline benchmark to any planar electromagnetic analysis, first capture a 50 Ohm stripline with the width indicated in Table I. Then make the relative dielectric constant unity and set the ground plane spacing to 1.0 mm. In air at 15 GHz, a quarter wavelength is 4.99654097 mm, use this value for the line length. Analyze the structure at 15 GHz and read the first order estimate of the error directly from the calculated S-parameters.

VI. A Specific Application of the Stripline Benchmark

We have applied the above stripline benchmark and error metric to a commercial electromagnetic analysis [5], [6]. Error was investigated as a function of two subsectioning parameters, N_W and N_L. N_W is the number of cells per line width and N_L is the number of subsections per wavelength along the line length. We have found that the error is well represented by the following expression:

$$e_T \leq \frac{16}{N_W} + 2\left(\frac{16}{N_L}\right)^2 \qquad N_W > 3 \quad N_L > 15$$

where e_T is total error in percent, N_W is the number of cells per line width and N_L is the number of cells per wavelength. N_W is present to the inverse first power due to the step approximation of transverse current. N_L is present to the inverse second power due to the piece-wise linear representation of longitudinal current.

We expect this expression for error to be valid for the error due to subsectioning in any analysis which uses rooftop subsections. Of course, other error sources, as described above, may cause the actual error for a particular analysis to exceed this value. When present, these additional error sources should also ideally be included in the error expression.

A surprising result is that, except for small values of N_W, the error is independent of the actual line width. The error depends only on the number of cells into which the line width is divided. A 50 Ohm line with 32 cells across the line width has the same percent error as a 100 Ohm line with 32 cells across the line width. While the above expression was derived from stripline data based on absolute convergence, it also works well for microstrip (based on relative convergence results).

The stripline benchmark has been applied to lines with N_W up to 512 cells per line width and N_L up to 512 cells per wavelength along the line length. Execution time on an HP-710 at the highest level of resolution and accuracy is 1969 Seconds with a total error of 0.05%. With most published results at the level of $N_W = 1$ to 3, we feel this represents state-of-the-art for electromagnetic analysis. At $N_W = 1$ to 3, execution time is less than one second, however the total error has climbed to about 6%.

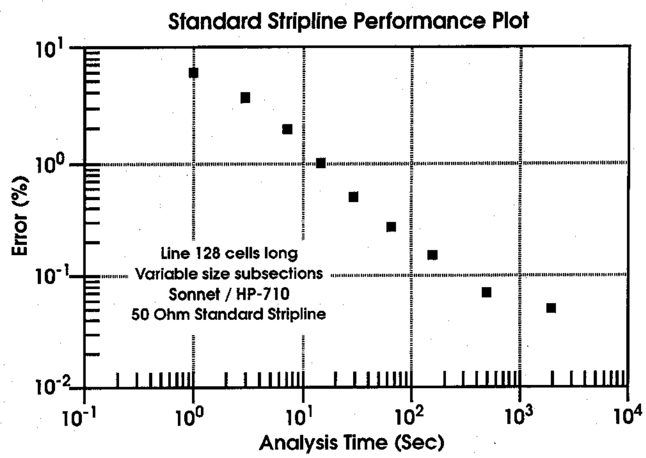

Fig. 2. As subsection size is reduced, longer analysis times result. In exchange for the longer analysis times, reduced error is realized. We see absolute convergence to 0.05% as subsection size is decreased.

Error versus analysis time is shown in Fig. 2. We call this a "performance plot." This particular plot uses the first order error metric (based directly on the S-parameters of a quarter wavelength line) to allow other researchers to easily duplicate and compare results. The primary application of the performance plot is to check for absolute convergence. For Fig. 2, we set the subsection size to 128 cells along the length of the line (512/wavelength) and varied the cells per line width from 2 to 512 per width. Variable subsection size (in terms of cells) was used. It was determined that the error introduced by this kind of subsectioning is less than 0.01%.

In Fig. 2, we see strong, uniform convergence down to the 0.05% error level. At this point it appears to be approaching a limit for the given subsectioning parameters. Thus we can conclude that, at least for this case, all error sources detectable by this benchmark are less than 0.05%. It is not unreasonable to expect that most of the residual 0.05% error is due to subsectioning.

This plot explicitly, quantitatively, and precisely shows the trade-off between analysis time and accuracy. It can be used to compare the same analysis on different computers. It can also be used to compare different analysis techniques on the same computer. As different researchers naturally use different computers, this may be difficult. As more benchmarks become available, separate performance plots should be generated for each benchmark. A characteristic of the performance plot is that, all else being equal, performance curves lower and to the left are always better.

In most engineering applications, error on the order of 10% is generally not useful. To achieve "Success on first fabrication," analysis error significantly less than manufacturing tolerances is required. In this case, error on the order of 1% or better is useful. In rare cases, error on the order of 0.1% is desired. For this level of accuracy, as shown in Fig. 2, analysis time grows quickly.

We suggest that presentation of an equation, as above, specifying an upper bound on the error as a function of discretization parameters and a performance plot, such as Fig. 2, showing error versus execution time, should both become a expected part of generally accepted validation procedures. This is in addition to the more common GABMAC validation approaches. Over time, we hope to see additional benchmarks which allow the detection and quantification of error sources not included in this initial benchmark.

VII. An Example of the Impact of Analysis Error

Many electromagnetic analyses are optimized for the use of small values of N_W. For example, a 50 Ohm line may be subsectioned one cell wide, $N_W = 1$. In this case, we find the subsectioning error to be about 5% to 6%. In some applications, this might be acceptable without realizing its true implications. In this section we show an example where this degree of error could be considered undesirable.

Fig. 3 shows an example of error due to subsectioning 1 cell wide. A single resonator filter [7], [8] can be analyzed with Sonnet at 10 cells per line width. Because of diagonal edges, Sonnet cannot presently analyze the structure at one cell wide. However, other software was used [8] to analyze the filter at one cell wide resulting in the curve labeled PMESH in Fig. 3. The one cell wide approximation appears to result in underestimation of both insertion loss and bandwidth. This may be

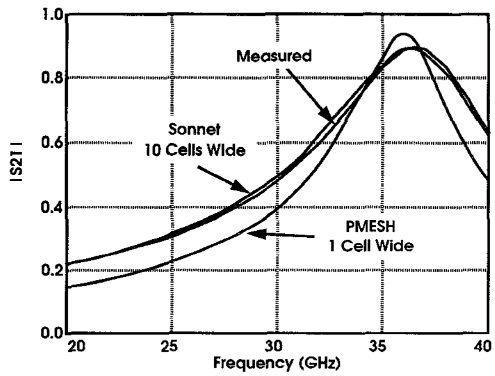

Fig. 3. Measured and calculated data for a diagonal single resonator bandpass filter. The effect of about 5% error is indicated by the curve labeled "PMESH." This plot should be used to compare the effect of subsectioning only. It should not be used to compare the analyses.

due to the actual edge singularity in the current distribution being approximated with a uniform current. Keep in mind that PMESH has not been rigorously validated according to the techniques described in this paper. Thus we cannot strictly rule out other error sources, for example de-embedding error, as the cause of the discrepancy shown in Fig. 3.

In this case we are unable to perform both the 1 cell wide and the 10 cell wide cases with one analysis. We expect that any roof-top subsection based electromagnetic analysis capable of performing either case should give similar results for each such case. Fig. 3 is intended to compare the effect of different subsection sizes. It is not intended to compare different analyses.

If error present in the 1 cell wide case is acceptable in a given design (for example, if only the resonant frequency were desired), then the one cell wide approximation is appropriate. There is no need for the increased analysis time caused by higher resolution. If more accuracy is needed, a smaller cell size should be invoked.

VIII. Conclusion

We have described an error metric and a precise benchmark. The benchmark is accurate to a level of about 10^{-8}. A simple means of determining the residual error of any planar electromagnetic analysis by using a special case of the benchmark has been described. The benchmark has been illustrated by application to an existing electromagnetic analysis. As a result, an explicit equation for the residual error of the analysis has been developed. The benchmark may be applied to any planar electromagnetic analysis capable of analyzing stripline to allow critical, quantitative, evaluation of accuracy.

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