A Conformal Mesh for Efficient Planar Electromagnetic Analysis

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***Abstract*—A planar electromagnetic analysis can provide faster analysis by using larger subsections at the cost of reduced accuracy. However, even if both rectangular and triangular subsections are used, large subsections are not practical for complicated curving planar circuits. This paper describes a method for joining small subsections so that the large subsections so formed can follow the arbitrarily curving edges of a complicated circuit while still inherently including the high edge current. Using such conformal subsections, non-Manhattan geometries can be analyzed efficiently and accurately. This is especially important for continuously curving geometries (like circular spiral inductors), which cannot be efficiently meshed using rectangular and triangular subsections. These conformal subsections retain nearly all the accuracy of small subsection size while also realizing the speed of large subsections, even for complicated geometries.**

***Index Terms*—Computer-aided design (CAD), conformal, edge effect, Electromagnetic (EM), fast Fourier transform (FFT), high frequency, mesh, method of moments (MoM), planar, rooftop, spiral inductor, subsection.**

Definitions

Cell	Elemental area of circuit metal.
Conformal subsection	Subsection whose area and current distribution conform to the (possibly curving) edge of the circuit metal.
Crossover string	String that allows current to flow from one mutual meeting point on an edge of a planar transmission line to another mutual meeting point, usually on the opposite edge of the planar transmission line.
Eave edge	End of a rooftop or conformal basis function where the current has decreased to zero.
Elemental subsection	Smallest subsection used to build larger subsections.
Funnel	Portion of a string that takes current from its normal path (usually longitudinal) and diverts it (usually transversely) to a mutual meeting point.
Merge	Specify that the current on one subsection is set to a fraction or to a multiple of another so that the number of degrees of freedom in the complete system decreases by one.
Mutual meeting point	Cell where multiple strings meet so that current may flow from any one string to any other string meeting at the same cell.
Open edge	Edge in a conformal subsection where maximum (edge effect) current flows.
Opposite edge	Edge of a subsection opposite the open edge.
Peak	Locus of highest current in a roof top subsection or the high current ends of all the strings in a conformal subsection.
String	Group of cells following a single, simple, and complete path between two points.
Subsection	One or more basis functions (typically rooftops) that together represent a single degree of freedom for the electromagnetic (EM) solution.
Weight	When one subsection is merged with another to form a larger subsection, it is assigned a current that is a fraction or a multiple of the other. This multiple is the weight of the subsection.

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I. Introduction

IT IS WELL known that analysis time for planar EM analysis using method of moments (MoM) increases $O(N^3)$, where N is the number of subsections. While using various iterative matrix solvers can yield a faster analysis, such solutions entail a reduction of robustness and accuracy that is undesirable for some applications.

Given that an $O(N^3)$ matrix solver is used, a common approach to realizing faster analysis is to reduce N by merging small subsections into a single large subsection, as shown in Fig. 1. Now, instead of increasing with $O(N^3)$, analysis time decreases with $O(N^3)$. Even a small reduction in N can yield significantly faster analysis.

Present-day MoM EM analyses can handle up to 20 000 (double precision, lossless) or 30 000 (single precision, lossless) subsections in about 1 h per frequency on a 3-GHz class computer. If a 20 000-subsection analysis can be reduced to 2000 subsections, analysis time is reduced to seconds per frequency. Analyses that would have previously required 200 000

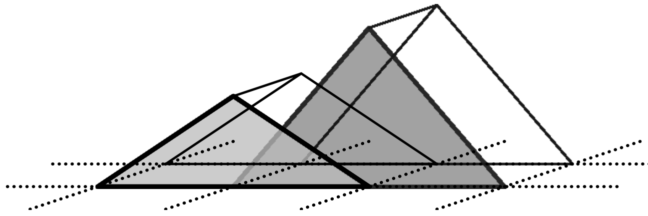

Fig. 1. Each rooftop basis function covers two cells. Two rooftops can be merged together, with appropriate weighting, to yield a larger rooftop, covering three cells. Vertical dimension indicates current density.

subsections suddenly become viable at the 20 000-subsection level.

Rooftop functions [1] are a common basis function. When using a fast Fourier transform (FFT)-based EM analysis of shielded circuits [2], it is most convenient to use only one size of rooftop. This minimizes the required number of FFTs. A reduction in N can be obtained by merging small elemental rooftops into larger rooftop basis functions where possible, as shown in Fig. 1. If we restrict the large rooftops at the edge of lines to a narrow width, a representation of the edge effect or the high current at the edge of the line is retained. This is important for accuracy.

It is possible to form more arbitrary subsections including triangles [3] and even polygons [4]. However, since the edge effect is not inherently included in these basis functions, there must still be narrow subsections along the edge of circuit metal or accuracy degrades, especially for loss calculations.

Even more arbitrary subsections can be formed by merging many small basis functions into larger subsections [5]. These subsections include the edge effect, conform to the curving edges of transmission lines, and can be multiple wavelengths long. However, generating such a mesh requires at least one complete EM analysis for each significant pair of subsections and this can become a lengthy process as the number of subsections grows. In addition, this process must be repeated for each frequency. When subsections are different at any frequency in an analysis, the resulting data can be discontinuous. This is undesirable for optimization and can compromise the performance of advanced interpolation approaches.

The conformal meshing described in this paper requires that subsection length be small compared to the shortest wavelength. However, no EM analysis is required in order to determine the subsectioning, exactly the same subsectioning is used at all frequencies, and the edge effect is inherently included.

II. Background

Merging small rooftops into larger rooftops is widely used to reduce subsection count. Fig. 1 shows two small rooftops, or "elemental subsections," merged in this manner. Each elemental rooftop covers two cells. Since the rooftops overlap, the sum of the two rooftops covers three cells. The current on one rooftop is set to twice the current on the other rooftop, reducing the previous two degrees of freedom to one. This amounts to adding appropriately weighted rows and columns in the MoM matrix. The result is one larger rooftop with two cells to one side of the peak and one cell on the other side of the peak.

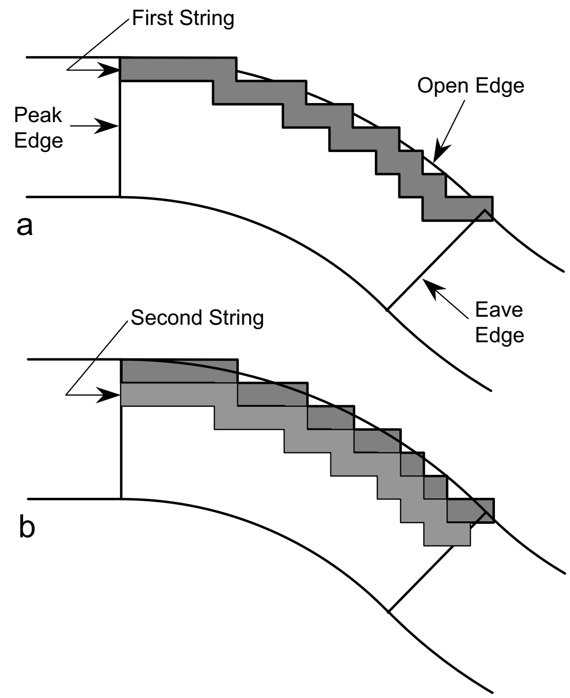

Fig. 2. Conformal meshing considers a section starting with the "peak edge" and ending with the "eave edge." One outside edge is considered the "open edge," where the high edge current flows. (a) The path for the first string goes along this edge. (b) A second string is added.

As long as the resulting larger rooftop covers a rectangular area and the surface on both sides of the peak is everywhere exactly linear, then an arbitrary number of elemental rooftops can be merged into a larger rooftop. When multiple larger rooftops are formed from elemental rooftops, then current flows properly from one to the next as long as the larger rooftops are overlapped in the same manner as the original elemental rooftops. Even the slightest failure to overlap the large rooftops properly results in an open circuit for the offending region.

Provided narrow subsections are maintained on all metal edges, this sort of subsection merging works well for rectangular, or "Manhattan" geometries. While still accurate, it is not as efficient for nonrectangular geometries. Conformal meshing addresses this problem.

III. Conformal Meshing

Conformal meshing consists of the assignment of weights to all the small elemental rooftop basis functions in a given region. If this assignment is not done properly, then that region becomes an open circuit in the analysis. Our approach is a generalization of the rectangular rooftop to an arbitrary area in a manner that includes the edge effect [6].

Illustrated in Fig. 2(a), a curved transmission line is conformally meshed by taking a section of the transmission line starting with a "peak edge" and ending with an "eave edge." The first string is a path along the open edge as shown. A cell (as defined by an underlying uniform grid, not shown) is included in the string if its center falls inside the open edge. The string covers the path from the peak edge to the eave edge by following the open edge.

Merging elemental rooftop subsections so as to cover the entire string from peak to eave initiates the formation of the conformal subsection. Each elemental rooftop overlaps, as shown in Fig. 1. Both X- and Y-directed rooftops are added as needed to make a complete path from peak to eave.

The first elemental rooftop (at the peak edge) is given a weight of 1.0. Subsequent rooftops are given linearly reduced weight so that the last elemental rooftop, at the eave edge, is exactly zero. All weights must taper exactly linearly between peak and eave. This is critical. As with the merged rectangular rooftops of Fig. 1, if there is anything other than an exact linear taper from peak to eave, the given region of metal becomes an open circuit.

Next, a second string is added adjacent to the first. Note that the elemental rooftops on the second string overlap the first string at certain turns. In order to include the edge effect, the first elemental rooftop of the second string is given a reduced weight compared to the first string. The further a string is from the open edge, the smaller the weight. The weights along the length of every string are tapered exactly linearly starting with the weight assigned to the peak elemental rooftop and then going down to zero at the eave.

This procedure is repeated until the entire area of the transmission-line section, between the peak and eave, is covered with strings. All the strings so formed are merged and constitute the first of four conformal subsections that typically cover a given area.

In analogy with overlapping rooftop subsections, a second conformal subsection is added on exactly the same area as the first conformal subsection. The second subsection is identical to the first subsection, except the peak and eave edges are swapped. In Fig. 2, the first subsection has strings that peak along the left end and linearly taper to zero on the right end. The second subsection has strings that peak on the right end and exactly linearly taper to zero on the left end. These two conformal subsections exactly overlap the same area in exactly the same fashion as two normal rooftop subsections overlap. With exactly identical, but oppositely sloping linear tapers, current can flow from the peak edge of one subsection to the peak edge of the other subsection.

Recall that the open edge string has the highest current, modeling the edge effect. Interior strings have successively less current. The current flowing on strings that start to approach the opposite open edge could be assigned higher weight to model the edge effect on the opposite edge. This is appropriate when the proportion of current flowing on the opposite edge as compared with the open edge is known.

Generally, this is not known. Instead, the string on the opposite edge is given the smallest weight of all; there is no consideration in the string weight for the proximity of the opposite edge. Rather, a third conformal subsection is added, exactly like the first, only with the open edge (as labeled in Fig. 2) and the opposite edge swapped. Finally, a fourth conformal subsection is added, exactly like the third, only with the peak and eave edges swapped.

These four conformal subsections allow current to flow along either edge of the transmission line from the peak edge of one conformal subsection at one end to the peak edge of its mating conformal subsection at the other end.

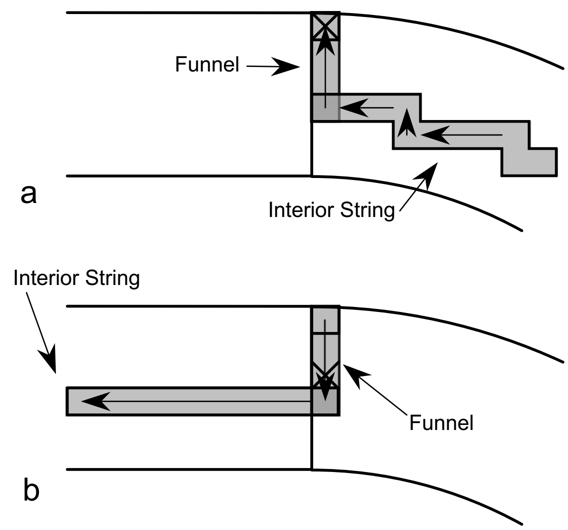

Fig. 3. (a) String on the interior of a conformal subsection allows current to flow longitudinally to the peak edge (center). From here, the current flows in a "funnel" string to the mutual meeting point indicated by the "X." (b) In the adjacent conformal subsection, a string draws current from the mutual meeting point along its own funnel string.

As described above, each string in each conformal subsection is linearly tapered longitudinally along its length. Due to the addition of the third and fourth subsections, each conformal subsection must also be linearly tapered to zero across its width, transversely. This transverse linear taper multiplies the string-to-string taper already described, which models the edge effect. In this way, each subsection has no influence on its opposite edge and has complete control of its own open edge.

IV. Connecting Adjacent Conformal Subsections

A normal rooftop subsection (Fig. 1) has one peak and two eaves. The conformal subsections thus far described each have one peak, but only one eave. It is possible to add a second eave on the other side of the peak to complete the conformal analogy with the normal rooftop basis function. For reasons to be explained, this is not done.

To complete the conformal subsectioning, the metal of the entire circuit is divided into regions similar to the region (between peak and eave) shown in Fig. 2. The above conformal subsectioning algorithm is then applied to all such regions.

Note the peak edge as labeled in Fig. 2. This is the peak edge for the first and third conformal subsections described above. These subsections are immediately to the right of the indicated peak edge. When the conformal meshing algorithm is applied to the region immediately to the left of the indicated peak edge, the second and fourth subsections generated also use this same peak edge.

The problem now becomes how to allow flow of current from the conformal subsections on the right to the conformal subsections on the left, all of which share the same peak edge. The solution is illustrated in Fig. 3.

All current from all strings is funneled to a mutual meeting point. A typical interior string and its associated funnel is illustrated in Fig. 3(a). All strings meet at the same mutual meeting point, indicated by the "X" in Fig. 3. The weight of all the rooftops in a given funnel string is the same as the weight of the peak elemental rooftop in the string being funneled.

A typical interior string in the adjacent subsection is illustrated in Fig. 3(b). The funnel string takes current from the mutual meeting point and provides it to the interior string. There is one funnel string for each interior string. All the funnels should be restricted to the same path.

Typically, all the funnel strings for both subsections are all merged with one of the two subsections, with appropriate normalization. Notice that the current direction for the funnel string in Fig. 3(b) is opposite that of the funnel in Fig. 3(a). Thus, if there are the same number of strings in both subsections, and the string weights are distributed identically, then when all the funnels of both subsections are merged there will be zero total current in the funnels. In other words, current can flow from one subsection directly to the next without the assistance of funnels.

However, if one subsection has a different number of strings, or if the string weights are distributed differently, then one or more total funnel elemental rooftop weights are nonzero. In this case, the elemental rooftops added by the funnels are necessary for current to flow from one subsection to the next. If the funnels are not added exactly correctly, an open circuit between the two subsections forms.

V. Crossover Current

When flowing around a bend, or when encountering other discontinuities, current may need to flow from one side of a line to the other. While it is possible to add area covering subsections to allow this current, such crossover current is usually small and can be modeled more efficiently by means of a "crossover string."

A mutual meeting point is shown in Fig. 3. This meeting point is on the open edge of the line and allows current to flow across the peak edge from one subsection to the next. Recall that there are also two more conformal subsections that allow current to flow along the opposite open edge of the line. These two subsections also have a mutual meeting point, on the opposite end of the same peak edge.

A crossover string connects these two mutual meeting points. Thus, the crossover string carries current along the peak edge from one side of the transmission line transversely to the other side. Each elemental rooftop in the crossover string has the same weight. All the elemental rooftops in the crossover string are merged together. The entire crossover string is treated as a single subsection. It is not merged with any other subsection. Now current can switch, as needed, from side to side by flowing through the crossover string.

For higher accuracy, the crossover string can be added as two overlapping linearly tapered strings. In this way, the crossover current can vary linearly as it flows from one mutual meeting point to the next. This is the approach we use.

Note that we cannot include crossover strings in this way if the two conformal subsections on either side of the peak edge were merged together into a single subsection. If this were done, then Kirchoff's law could be met at the mutual meeting point only by assigning zero current to the crossover string. With the total longitudinal current free to vary independently on either side of the peak edge, then crossover current can flow as needed.

Transverse current is important at discontinuity boundaries. The algorithm for selecting peak edges should always place edges at all discontinuities so that crossover current can flow.

Note that, created as specified above, the peak-to-eave strings carry the longitudinal current and the crossover strings carry the transverse current flowing on a transmission line. Since transverse current is limited to crossover strings, this conformal meshing should be used only where transverse current is a fraction of the longitudinal current. It should not be used on large-area structures, like patch antennas, where the distinction between transverse and longitudinal current is not clear from the geometry.

VI. Edge Effect Current

For a microstrip line along the x-direction, the current distribution across the width can be approximated by [7]

$$J_X = \frac{1 - A\left(\frac{2y}{w}\right)^2}{\sqrt{1 - \left(\frac{2y}{w}\right)^2}} \tag{1}$$

where

J_X current density;
y distance from center of line;
w width of the line;
A scale factor.

Integrating over the width of a single elemental rooftop yields the total current on one cell width

$$I_X = \left[\left(1 - \frac{A}{2}\right)\arcsin\left(\frac{2y}{w}\right) + \frac{A}{2}\left(\frac{2y}{w}\right)\sqrt{1 - \left(\frac{2y}{w}\right)^2}\right]_{Y_0}^{Y_0+\Delta Y}. \tag{2}$$

We use this expression, multiplied by a linear taper (as mentioned previously) to assign the weight for each string as a decreasing function of its distance from the edge of the transmission line.

While [7] gives an expression for A, it is a function of frequency. In addition, the expression is intended only for microstrip. As already mentioned, the subsections must remain unchanged for analysis at all frequencies of interest, and possible geometries are not limited to microstrip. Thus, we use an approximate value for A. To reduce error caused by this, the edge string of each subsection is formed into a separate subsection. In this way, the edge current, which is the most important part of the current distribution, is free to vary with frequency and geometry. In situations where reduction of subsection count is especially important and the reduced accuracy of not having a precise value for the edge current is acceptable, the edge string may be merged with the rest of the subsection.

Alternatively, two subsections could be added in place of each conformal subsection using (2). One subsection includes the string weight terms above, which are not multiplied by A. The other subsection includes all string weight terms that are multiplied by A. With this modification, the factor A becomes a degree of freedom. However, matrix fill time increases due to the increased subsection complexity.

VII. Other Considerations

In Fig. 2, the region being subsectioned is bounded by two internal (peak/eave) edges. At junctions of multiple transmission lines, the junction region is bounded by more than two internal edges. In this case, adjacent pairs of internal edges are taken and conformally subsectioned one pair at a time.

After extensive testing, we found that the conformal meshing accuracy can occasionally degrade at such multiple transmission-line junctions. Since robustness is an important requirement, we now revert all such junctions to regular rooftop subsectioning. A small area around vias is also automatically reverted to regular subsectioning to allow a reliable connection to the vias.

Some regions, like open ends, are bounded by a single internal edge. While such regions can be conformally meshed, we found the accuracy was not suitable for resonant structures. Therefore, the area of all such regions is kept small and reverted to normal rooftop subsectioning. To assure good connections between regions of conformal subsectioning and regular rooftop subsectioning, the bordering funnel strings are reverted to normal subsectioning.

Internal edges should be placed at all discontinuities so that crossover current can flow, as described above. Included in this category of discontinuities are air bridges and other crossovers. Even though both transmission lines might be uniform, current distribution can change dramatically in each where they cross over the other.

Extensive effort has been devoted to developing algorithms for selecting the internal peak/eave edges. Ideally, the internal edges should be short and transverse to the current flow. For efficient subsectioning, care should be taken that two internal edges are not redundantly placed close to each other. In addition, the internal edges cannot be allowed to cross over each other although multiple internal edges may share a single vertex.

Once internal edges are selected, a recursive algorithm selects the string paths. These algorithms are similar to maze solving routines. Care must be taken, for example, to back out of dead-end paths, to function properly even for zero length paths, and to select a path that does not form an infinite loop or exit the desired region. For example, when a region narrows, the path of an internal string may cross over the opposite edge. If the opposite edge is about to be crossed, the path of the string must be diverted to stay inside all the while proceeding toward the eave.

A special difficulty is in handling the case where cell centers fall exactly on one or more region boundaries. Extensive testing is required to identify and remedy numerous rare, but important, degenerate situations.

Once the conformal subsections are specified, then the list of elemental rooftops must be scanned for duplications. If two elemental rooftops are at the same location and are part of the same conformal subsection, they should be merged into one elemental rooftop with the sum of the original weights. If there is a subsection containing only a single elemental rooftop, then all other elemental rooftops at the same location can be removed. If a conformal subsection has no elemental rooftops or if an elemental rooftop has zero weight, it should be removed. Likewise, if one conformal subsection is identical to another, then one of the two must be removed. This last situation is common for transmission lines between one and three cells wide. Performing these and other related tasks efficiently for lists of 1 million or more elemental rooftops is challenging.

Many of the considerations and procedures described in this paper are easily understood visually, but are difficult to automate in a computer algorithm where one no longer has a picture to view, but rather must work only with arrays of numbers.

This type of conformal subsection is generally best used only on non-Manhattan geometries. Due to the complicated weighting of the elemental rooftops, MoM matrix fill takes longer. However, the reduction in total subsection count can be dramatic, yielding a much smaller MoM matrix and faster analysis. Once the subsection count is reduced below 5000 or so, matrix solve is extremely fast and conformal meshing should not be extended to any remaining circuit metal.

However, if the subsection count must still be further reduced, then conformal meshing can be invoked on the Manhattan portions of a circuit, keeping in mind that matrix fill takes longer.

VIII. Validation

A. *Exact Standard Stripline*

The conformal meshing algorithm has been validated on a regression test of over 1500 circuits that have been accumulated over 20 years for the express purpose of finding problems with EM analysis. This degree of testing is critical for an algorithm of this complexity. There are a large number of low-probability situations, some of which have been mentioned in the previous section, which can generate undesired results. The situations elicited by the extensive regression testing have all been identified and corrected, yielding a high degree of robustness that would not have been otherwise possible.

An important aspect of regression testing is the precise quantitative evaluation of the error performance of an EM analysis. For this purpose, we use the stripline standard [8]. The stripline is exactly 50 Ω and 1/4-wavelength long at 15 GHz. Results are summarized in Table I. N_W is the number of cells across the width of the line. For Table I, the line is 128 cells long, yielding 512 cells per wavelength. The Z_0 error decreases to 0.3% at $N_W = 128$. For larger N_W, the error remains constant. This is due to using an assumed value for A in (1). Z_0 error from 1% to 2% (N_W = 8–16) is sufficient for most applications. When less than 0.3% Z_0 error is needed, regular meshing should be used. Velocity of propagation error is negligible in nearly all cases.

Table II shows how velocity of propagation error changes with N_L, the number of cells per wavelength. N_W is set to 16. The first line of this table is the same data as the $N_W = 16$ line of Table I. While the velocity of propagation error is negligible, note that the Z_0 error starts decreasing as cell length becomes large, reaching a minimum at $N_L = 32$. This is an example of velocity error canceling Z_0 error. In fact, a value of N_L could be

TABLE I
STRIPLINE STANDARD ERROR VERSUS N_W

N_W	Z_0 Error (%)	Velocity Error (%)
2	5.3	0.70
4	3.4	0.05
8	2.0	0.00
16	1.0	0.00
32	0.6	0.00
64	0.4	0.00
128	0.3	0.00

TABLE II
STRIPLINE STANDARD ERROR VERSUS N_L

N_L	Velocity Error (%)	Z_0 Error (%)
512	0.00	1.0
256	0.00	1.0
128	0.00	1.0
64	0.01	0.9
32	0.11	0.5
16	0.19	0.9

selected that takes the Z_0 error nearly to zero. This error cancellation mechanism cannot be used in practice because it is sensitive to the specific cell size dimensions. If only select data were presented, one could be left with an incorrectly optimistic impression of the error performance.

In most of the standard stripline analyses, the conformal subsection size is the same, only the underlying cell size is changed. There are 26 conformal subsections along the length of the line, except when this is not possible due to large cell size. When the cell size is made smaller, the elemental rooftops become smaller and the conformal subsection becomes more accurate.

A common misperception is that cell size small compared to wavelength is sufficient to assure low error. This is usually true for the length of cells. However, when linewidth is already small with respect to wavelength, cell width must additionally be made small with respect to the width of the line, independent of wavelength [8]. When metal thickness is modeled and is already small with respect to wavelength, cell thickness must be made small with respect to the metal thickness, again, independent of wavelength [9].

B. Curved Transmission Lines

While the exact solution is known for the stripline standard, it provides no information as to how conformal meshing performs for curved transmission lines. For this reason, we devised an analysis of two curved lines, one uses conformal subsectioning (Fig. 4, left), the other uses the usual rooftop subsectioning (Fig. 4, right). The lines are 508-μm wide on a 254-μm-thick substrate with a relative dielectric constant of 10.0. There are

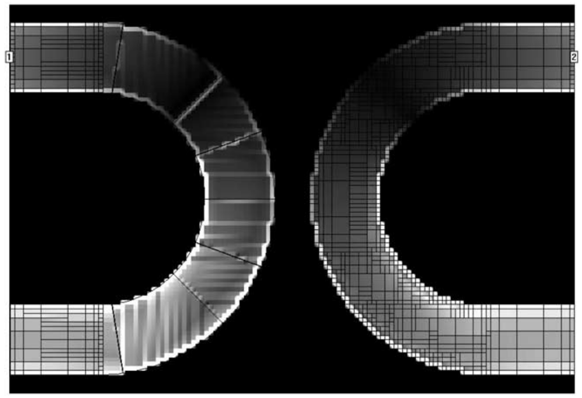

Fig. 4. With normal subsectioning on the right, we can see the dramatic reduction in subsection count provided by conformal subsectioning on the left. The general nature of both current distributions is the same, especially in regard to the edge effect.

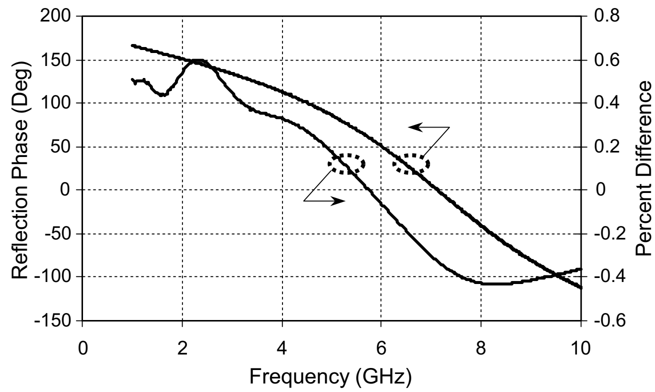

Fig. 5. Reflection phase results for both curved, shorted stubs in Fig. 4, are visually identical. Percent difference is under 0.6% at all frequencies.

16 cells across the linewidth. Only the curved portion of the left line is conformally meshed. Notice the substantial reduction in the number of subsections.

The nature and magnitude of the two current distributions (shown at 10 GHz) is nearly identical, especially in respect to the critical edge effect. However, the conformal meshing current distribution shows a "crystalline" structure. This happens where two strings overlap at turns, as illustrated in Fig. 2(b). There is more current where two strings momentarily overlap. Also, there are some transverse lines in the conformal current density. This is due to the crossover strings allowing current to flow from one side to the other.

Both circular transmission lines in Fig. 4 are shorted stubs. Thus, all the analysis errors in each stub affect the reflection phase. Fig. 5 shows the calculated reflection phase of both stubs as visually identical. For that reason, another curve is added, plotting the percent difference between the two results. The difference is less than 0.6% at all frequencies. The Sonnet adaptive band synthesis (ABS) interpolation required analysis at only eight frequencies to generate the entire 361-frequency data set.

C. Spiral Inductor

A large spiral inductor illustrates the use of conformal meshing in practice. A circular spiral inductor cannot be

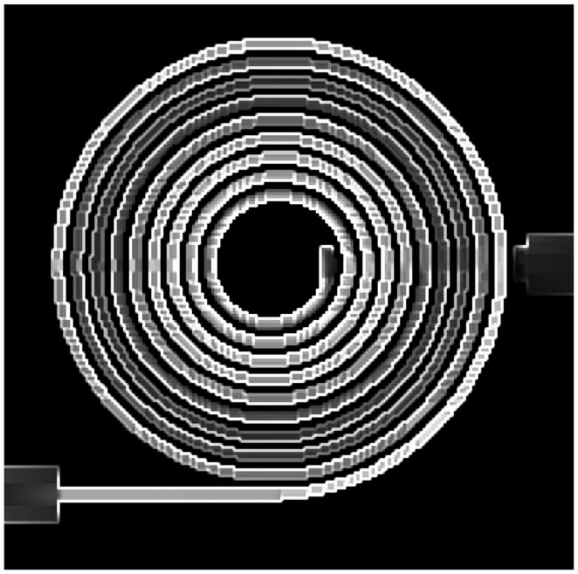

Fig. 6. Large spiral inductor on silicon shows how the edge effect current switches back and forth between sides as it flows around the spiral, illustrating the critical importance of including the edge effect when analyzing loss.

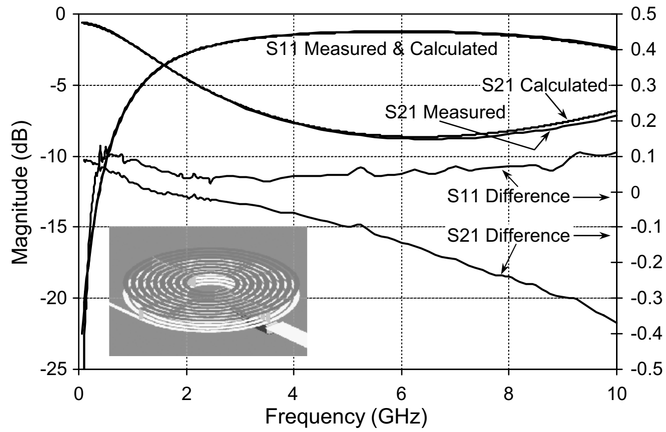

Fig. 7. Measured versus calculated for the spiral inductor on GaAs. The difference between the two is plotted on the right-hand-side axis.

efficiently subsectioned even when using arbitrary-size triangle subsections, especially when the edge effect must be included for accurate calculation of loss. This spiral, courtesy of Motorola, Tempe, AZ, uses Motorola's high-voltage integrated circuit (HVIC) Si RF-LDMOS process on 90-μm-thick high-conductivity silicon dielectric constant 11.9. Certain details of the dielectric stack up are proprietary and are not reported here. The lines are 6.5-μm wide with a 3.0-μm gap, and there are four cells across the linewidth. Metal thickness is 3.6 μm, bulk conductivity is 2.78×10^7 S/m. A two-sheet model [9] is used for the thick conductor.

Fig. 6 shows the current distribution on this spiral inductor at 10 GHz. Note that the high edge current often flows only on one side of the line, switching sides several times along the length of the spiral line. This is known as "current crowding" and is caused by the inductor's magnetic field penetrating the plane of the inductor. Current crowding illustrates why proper modeling of the edge effect is critical for accurate analysis of loss. If some way could be found to design an inductor so that the high edge current flows on both sides through the entire inductor, loss could be substantially reduced. One approach might be to split the spiral line in two along its length, and then swapping each side with the other periodically.

Fig. 7 shows the measured versus calculated results as nearly visually identical. For this reason, two additional curves show the difference between measured and calculated. Reflection differences are nearly everywhere under 0.1 dB, while transmission differences increase to nearly 0.4 dB at high frequency. The geometry that was analyzed is exactly the geometry as was provided by Motorola. There were no "tuning" modifications made to loss, dielectric stackup, dimensions, etc. The inset in Fig. 7 shows the spiral geometry (vertically expanded).

This spiral inductor requires 3238 subsections and 5 m 48 s per frequency on a 3-GHz P4. A total of six frequencies are required by the ABS interpolation to yield the entire data set of 397 frequencies. If it were important to reduce subsection count further, then the Manhattan portions of this circuit could also be converted to conformal subsectioning. However, inversion of a matrix of just a few thousand subsections is very fast.

To perform a convergence analysis, the cell size was cut in half (for four small cells taking the place of each original cell), and then cut in half again (now 16 small cells, taking the place of each original cell. Analyses at 10 GHz show maximum differences between all results and are less than 0.08 dB and 0.6°. The second of these two analyses is of particular note, as it consists of 1.7-million elemental subsections, perhaps the largest circuit ever analyzed using a noniterative MoM code.

Using regular subsectioning at the original mesh size requires 29 677 subsections and an estimated 7 GB of memory (lossy double precision), a problem size, which is simply not viable. An attempt to analyze this spiral using another EM tool that allows arbitrary triangle subsections for the interior of the line and narrow rectangles for the edge effect yields similarly untenable statistics.

IX. Conclusion

We have described the implementation and validation of conformal meshing suitable for the efficient EM analysis of planar circuits. Use of conformal meshing substantially reduces subsection count for complicated circuits. Since the conformal subsections inherently include the high edge current, analysis accuracy is nearly as good as using regular meshing. Conformal meshing now allows non-Manhattan circuits, especially those including curving transmission lines, to be analyzed both accurately and efficiently. This was not previously possible, even when meshing with arbitrary rectangles and triangles. The accuracy has been verified in over 1500 circuits. Presented here are results from the exact stripline standard, a comparison of regular and conformal meshing for a curved transmission line, and a large circular spiral inductor with measured data.

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