An Electromagnetic Time-Harmonic Analysis of Shielded Microstrip Circuits

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Abstract —**A Galerkin analysis of microstrip circuits of arbitrary planar geometry enclosed in a rectangular conducting box is described. The technique entails a time-harmonic electromagnetic analysis evaluating all fields and surface currents. This analysis is suitable for the accurate verification of microstrip designs prior to fabrication.**

A computer program implementing the analysis has been written in Pascal on a personal computer. Agreement with measurements of several microstrip structures suggests a high degree of accuracy.

I. Introduction

THIS PAPER DESCRIBES an electromagnetic analysis of arbitrary microstrip (i.e., planar) circuits contained in a rectangular conducting box. The analysis proceeds by subdividing the microstrip circuit metallization into small rectangular subsections. An explicit surface current distribution is assumed to exist in each subsection. We evaluate the tangential electric fields due to the current in each subsection and then adjust the magnitude of the current in all subsections such that the weighted residual of the total tangential electric field goes to zero on all metallization. All surface currents are determined and the problem is solved. The N-port circuit parameters follow immediately.

The assumed surface current distribution in a subsection is called an expansion function. With the integral of the resulting electric field weighted by the same function, we have a special case of the method of moments known as a Galerkin technique [2], [3]. The magnitude of the current in each subsection is "adjusted" by matrix inversion.

The fields due to current in an individual subsection are represented by a sum of homogeneous rectangular waveguide modes. Thus, this technique is closely related to the spectral domain approach [7]. The technique described here [4]–[6] was originally developed as an extension of an analysis of planar waveguide probes described in section 8-11 of [1].

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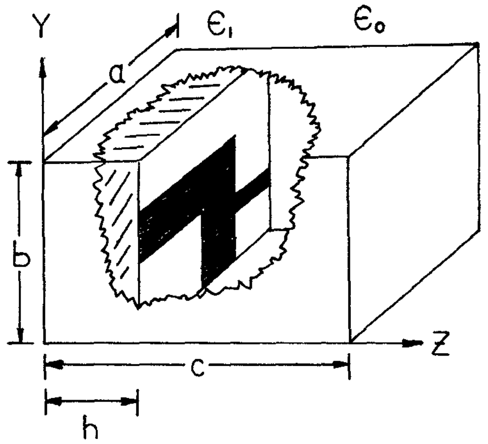

Fig. 1. The microstrip circuit is completely contained in a shielding, conducting rectangular box. The coordinate system is oriented so as to emphasize the fact that the fields are represented as a sum of homogeneous rectangular waveguide modes with the waveguide tube along the z axis.

II. Method of Analysis

The rectangular conducting box is treated as two separate waveguides joined at $z = h$ (Fig. 1) with the indicated regions and dielectric constants. Note that region 0 is usually but not necessarily restricted to free space. The tangential (or transverse to z) fields in a given region due to current on a single subsection is expressed as a sum of homogeneous waveguide modes. Expressions for the tangential fields are written as a weighted sum of these modes:

$$E_t^1 = \sum V_i \frac{\sin(K_{iz}^1 z)}{\sin(K_{iz}^1 h)} e_i$$

$$H_t^1 = \sum V_i Y_i^1 \frac{\cos(K_{iz}^1 z)}{\sin(K_{iz}^1 h)} h_i$$

$$H_t^1 = \sum V_i Y_i^1 \frac{\cos(K_{iz}^1 z)}{\sin(?K_{iz}^1 h)} h_i$$

$$E_t^0 = \sum V_i \frac{\sin[K_{iz}^0(c-z)]}{\sin[K_{iz}^0(c-h)]} e_i$$

$$H_t^0 = \sum V_i Y_i^0 \frac{\cos[K_{iz}^0(c-z)]}{\sin[K_{iz}^0(c-h)]} h_i \tag{1}$$

where V_i is the modal coefficient (amplitude) of the ith mode, and Y_i is the admittance of the ith (m, n) mode as follows:

$$Y_i^{1\text{TE}} = jK_{iz}^1/(\omega\mu_1) \qquad Y_i^{1\text{TM}} = j\omega\epsilon_1/K_{iz}^1$$
$$Y_i^{0\text{TE}} = -jK_{iz}^0/(\omega\mu_0) \qquad Y_i^{0\text{TM}} = -j\omega\epsilon_0/K_{iz}^0$$
$$K_{iz}^1 = -\sqrt{K_1^2 - K_x^2 - K_y^2} \qquad K_{iz}^0 = +\sqrt{K_0^2 - K_x^2 - K_y^2}.$$

Note that the modal admittances are the admittances of the standing wave modes rather than those of the usual traveling wave modes (they differ by the constant j). The $\boldsymbol{e}_i$ and $\boldsymbol{h}_i$ are the orthonormal mode vectors which form a basis for the expansion of the fields in each region. Note that the $m=0, n=0$ mode need not be included as all current is transverse to the z direction [8], [9]. For rectangular waveguide, we have

$$\boldsymbol{e}_i^{\text{TE}}(x, y) = N_1 g_1 \boldsymbol{u}_x - N_2 g_2 \boldsymbol{u}_y$$
$$\boldsymbol{e}_i^{\text{TM}}(x, y) = N_2 g_1 \boldsymbol{u}_x + N_1 g_2 \boldsymbol{u}_y$$
$$\boldsymbol{h}_i = \boldsymbol{u}_z \times \boldsymbol{e}_i \qquad \boldsymbol{e}_i = -\boldsymbol{u}_z \times \boldsymbol{h}_i$$

where

$$g_1 = \cos(K_x x)\sin(K_y y)$$
$$g_2 = \sin(K_x x)\cos(K_y y).$$

The N_1 and N_2 are normalizing constants dependent on the mode numbers and waveguide dimensions. If a different geometry is selected for the waveguide shield, only the above mode vectors need be changed.

Given a specific current distribution on the surface of the substrate, we must determine the modal coefficients, the V_i, of the field generated by that surface current. This is accomplished by setting the discontinuity in magnetic field equal to the assumed surface current. Then, using the orthogonality of the modal vectors, we may determine the V_i of the field generated by the current

$$V_i = -\hat{Z}_i \int\int \boldsymbol{J}_s \cdot \boldsymbol{e}_i \, ds$$
$$\hat{Y}_i = Y_i^0 \operatorname{ctn}\left[K_{iz}^0(c-h)\right] - Y_i^1 \operatorname{ctn}\left(K_{iz}^1 h\right)$$
$$\hat{Z}_i = 1/\hat{Y}_i.$$

The admittance $\hat{Y}_i$ is the parallel connection of the admittances of the two shorting planes at $z=0$ and $z=c$ transformed back to the substrate surface $z=h$. Multilayered geometries need only modify $\hat{Y}_i$.

Substitution of V_i into (1) yields the tangential fields everywhere in the waveguide. Specialization of V_i to a delta function for $\boldsymbol{J}_s$ provides the Green's function in the "spatial" domain for current on the surface of the substrate. The Green's function is a cosine and sine series in two dimensions with the coefficients of the series representing the Green's function in the "spectral" domain.

Evaluation of the V_i requires the evaluation of surface integrals of the current distribution dotted with a mode vector. We use the "rooftop" distribution [10], which is separable with respect to x and y. One component of current, either x or y, is evaluated at a time. The distribution has a triangle function dependence in the direction of current flow and a rectangle function dependence in the lateral direction. This is shown in Fig. 2, where the rectangular base of the three-dimensional figure represents the rectangular subsection, and the height above the base is proportional to current density. Fig. 3 shows how several rooftop functions can be placed on overlapping subsections to provide a piecewise linear approximation to the current in the direction of current flow and a step approximation in the lateral direction.

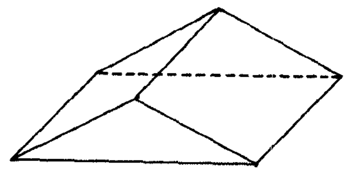

Fig. 2. The product of a triangle function in one direction by a rectangle function in the lateral direction gives a rooftop function which will be used as an expansion function.

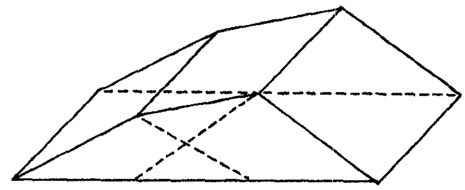

Fig. 3. Two rooftop functions placed on overlapping rectangles give a piecewise linear approximation to the current in the direction of current flow. Additional rooftop functions placed side by side will provide a step approximation to the surface current in the direction lateral to current flow.

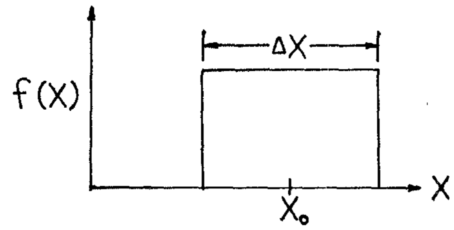

Fig. 4. The rectangular pulse function is used to represent the current density on a subsection in the direction lateral to current flow.

Since the rooftop function is separable, the integral for V_i reduces to the product of two one-dimensional integrals. The simplest integral to evaluate is the integral involving the rectangle function, Fig. 4. We require the evaluation of

$$F_c = \int f(x)\cos(Kx)\,dx \quad \text{and} \quad F_s = \int f(x)\sin(Kx)\,dx.$$

The constant K is the wavenumber corresponding to the variable of integration, for example, $M\pi/a$. Evaluation of the integrals yields

$$F_c = \frac{2}{K}\sin(K\Delta x/2)\cos(Kx_0), \qquad K \neq 0$$
$$F_s = \frac{2}{K}\sin(K\Delta x/2)\sin(Kx_0), \qquad K \neq 0$$
$$F_c = \Delta x \quad \text{and} \quad F_s = 0, \qquad K = 0.$$

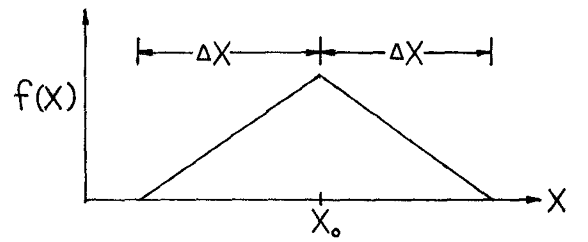

Fig. 5. The triangular pulse function represents the current density on a subsection in the direction of current flow. Thus, no line charges are generated.

Note that the term which depends on the subsection dimensions is the same for both cases. We refer to that term as $G(\Delta x)$. The integrand may be a function of y, in which case we have $G(\Delta y)$. For the rectangular pulse, we have

$$G(\Delta x) = \frac{2}{K}\sin(K\Delta x/2), \qquad K \neq 0$$
$$= \Delta x, \qquad K = 0.$$

We use F_c, F_s, $G(\Delta x)$, and $f(x)$ as a generic notation. The functions which they represent depend on the pulse being considered. For the triangle function, Fig. 5, we have

$$F_c = G(\Delta x)\cos(Kx_0) \quad \text{and} \quad F_s = G(\Delta x)\sin(Kx_0)$$

with

$$G(\Delta x) = \frac{2}{\Delta x K^2}(1 - \cos(K\Delta x)), \qquad K \neq 0$$
$$= \Delta x, \qquad K = 0.$$

Thus, to evaluate the integral for V_i we need only evaluate the ith modal vector at the midpoint of the subsection and multiply by the appropriate $G(\Delta x)$ and $G(\Delta y)$.

In the following equations, $f(x)$ denotes a pulse function which is a function of x, and $f(y)$ denotes a pulse function which is a function of y. $G(\Delta x)$ is the constant, derived above, which is obtained when $f(x)$ is multiplied by a sine or cosine and integrated over the domain of $f(x)$. The same holds for $G(\Delta y)$. We indicate distinct $f(x)$ and $G(\Delta x)$ functions by subscripts. Quantities relating to a source subsection are indicated by a prime. Quantities relating to a field subsection remain unprimed. A primed modal function indicates that the modal vector is to be evaluated at the center of the source subsection, e.g., $g_1' = g_1(x_0, y_0)$. An unprimed modal function is to be evaluated at the field point (or center of the field subsection).

In general, we consider a current distribution of the form $\boldsymbol{J}_s = J_x f_1(x) f_2(y)\boldsymbol{u}_x + J_y f_3(x) f_4(y)\boldsymbol{u}_y$. The pulse functions f_1 and f_4 are triangle functions while f_2 and f_3 are rectangle functions. Other functions could be used [4]. The pulse functions are centered on the subsection under consideration. Calculation of the V_i and substitution into (1) to calculate the tangential electric field at the substrate surface yield

$$E_x = \sum_{m,n}\left[-G_1'(\Delta x)G_2'(\Delta y)g_1'g_1\left(N_1^2\hat{Z}_{mn}^{\mathrm{TE}} + N_2^2\hat{Z}_{mn}^{\mathrm{TM}}\right)\right]J_x$$
$$+\left[G_3'(\Delta x)G_4'(\Delta y)N_1N_2g_2'g_1\left(\hat{Z}_{mn}^{\mathrm{TE}} - \hat{Z}_{mn}^{\mathrm{TM}}\right)\right]J_y$$
$$E_y = \sum_{m,n}\left[G_1'(\Delta x)G_2'(\Delta y)N_1N_2g_1'g_2\left(\hat{Z}_{mn}^{\mathrm{TE}} - \hat{Z}_{mn}^{\mathrm{TM}}\right)\right]J_x$$
$$+\left[-G_3'(\Delta x)G_4'(\Delta y)g_2'g_2\left(N_2^2\hat{Z}_{mn}^{\mathrm{TE}} + N_1^2\hat{Z}_{mn}^{\mathrm{TM}}\right)\right]J_y. \tag{2}$$

This equation is similar to (18) in [7], illustrating the similarity between this technique and the spectral-domain approach.

For a Galerkin implementation, we need the integral of the electric field weighted by a rooftop function, say $f_1(x)f_2(y)$, at a field subsection. This integration is effected by multiplying each term in the summation by $G_1(\Delta x)G_2(\Delta y)$. These weighted integrals (reactions) of the electric field are also used in the evaluation of the N-port circuit parameters.

A. Implementation of the Galerkin Solution

The technique is implemented by subdividing the metallization into small, overlapping rectangles. We need two sets of rectangles, one for x-directed and a second for y-directed current. The centers of the two sets of subsections need to be offset with respect to each other; otherwise the microstrip edges will not properly align. More importantly, a subsection of x-directed current cannot induce current in a collocated y-directed subsection. This situation gives incorrect results.

Both problems are solved by offsetting one set of subsections as in [10]. The current densities on the subsections form a set of dependent variables in a system of equations. The weighted integrals of the electric field on the subsections form a set of independent variables related to the dependent variables by an impedance matrix whose elements are calculated above. Select one (or more) subsections as a source; set the integral of the electric field on that subsection equal to one and all the others to zero (zero tangential electric field on a conductor). Matrix inversion provides the solution. Techniques for the efficient calculation of the matrix elements have been developed [4], [5].

B. The Source Model

Microstrip circuit inputs and outputs are usually taken at the edge of the substrate by means of a coaxial cable penetrating the shielding sidewall at $z = h$. The coax shield is connected to the microstrip shield, and the coax center conductor is attached to a microstrip conductor.

The coax aperture can be modeled by a conductor-backed circulating magnetic current. We assume that the aperture is small and that the aperture current has negligible effect. When we compare measured data with calculated data [4], [6], we find that the contribution from the aperture field is important and that it can be modeled as a small fringing capacitance in shunt with the connector.

We model the current injected by the coax center conductor as a subsection of current directed perpendicularly to the sidewall and centered on the sidewall. The port subsection uses the same roof-top current distribution. This facilitates the transition from the port subsection to microstrip subsections. In the analysis, we set the tangential electric field to a constant value on all port subsections and to zero on all other subsections.

C. Evaluation of Input Admittance

We initially discuss the input admittance of a one port circuit. Quantities associated with that port are designated by subscript 1. Elements in the admittance matrix of the entire microstrip system have double numerical subscripts.

We use the usual variational expression [1, pp. 348–349]

$$Y_i = -\frac{I_1^2}{\iint E \cdot J \, ds}.$$

Since $\boldsymbol{E}$ or $\boldsymbol{J}$ is zero everywhere except at the port subsection, we need only consider the port subsection. The weighted integral of the electric field on that subsection is equal to one by definition. The current on the subsection is proportional to that same weighting function, the constant of proportionality being $Y_{11} = J_1$. Thus, the denominator of the above expression is just Y_{11}. The input current is the input current density multiplied by the width of the input, Δw, usually either Δx or Δy. Thus,

$$Y_1 = -(Y_{11}\Delta w)^2 / Y_{11} = -Y_{11}(\Delta w)^2.$$

In a like manner, the transfer admittance between any two ports, say port a and port b, of an N-port circuit may be determined by

$$Y_{\text{tran}} = -Y_{ab}\Delta w_a \Delta w_b.$$

The sign of a transfer admittance depends on circuit geometry. This is because we define positive current in the direction of the positive axis, while circuit theory defines positive current as directed into the body of the multiport.

III. Software Implementation

The analysis was implemented in a Pascal program on an IBM-PC and later transported to a VAX computer. Dynamic arrays (a data type available in Pascal) were used extensively in developing a complex vector data type which was used to vectorize the software.

On the IBM-PC a small circuit (a dozen subsections) can be analyzed in a few minutes. Larger circuits (100 subsections) require several hours per frequency. The VAX version of the analysis provides a factor of ten improvement. The software has not been optimized.

A mouse-based microstrip geometry capture program has also been written. A five-section low-pass filter was subdivided into 611 subsections in less than an hour using this program. The output of the program (a text file containing the coordinates of the center of each subsection) is used directly as input to the analysis program.

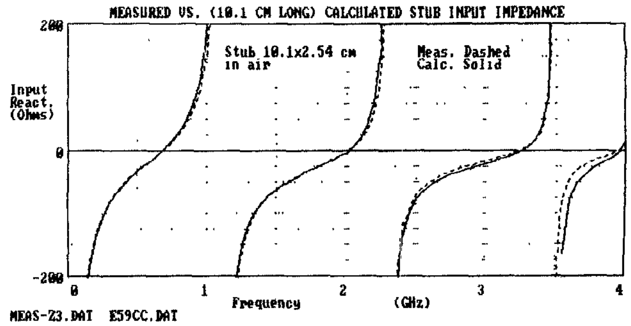

Fig. 6. Measurements (dashed line) of a large open-circuited microstrip stub agree well with calculations (solid line), suggesting that a high level of accuracy has been realized.

Fig. 6 shows a sample analysis, a comparison between measured and calculated data for a microstrip open circuited stub. The stub is 10.1 cm long and 2.54 cm wide. The agreement between the measured and calculated data is typical. Space does not permit more detailed results here. The interested reader is referred to [4] and [6].

IV. Conclusions

A technique for the analysis of shielded microstrip circuits has been presented. The technique is a Galerkin implementation of the method of moments and is closely related to the spectral-domain approach. The analysis is a complete time-harmonic electromagnetic analysis of microstrip.

The analysis may be used in the evaluation of individual microstrip discontinuities or, with faster computers, in the evaluation of entire microstrip circuits.

While the analysis is numerically intensive, it is sufficiently efficient that results for simple circuits can be obtained in reasonable time even with a small personal computer.

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