Testing Limits of Algorithms Associated with High Frequency Planar Electromagnetic Analysis

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Abstract — **The accuracy of numerical electromagnetic analysis is limited by numerous sources of error. Dynamic range is a fundamental limitation due to the finite precision arithmetic used by computers. This limitation, as well as limitations involving box resonances and subsectioning approaches, is explored quantitatively. In this way, users of such software can gain an understanding as to the range of problems for which a numerical electromagnetic analysis can give accurate answers, and the range for which the results may be in question.**

I. INTRODUCTION

Numerical electromagnetics research often devotes considerable effort to demonstrate the accuracy of an algorithm. This paper takes the opposite approach. We explore the error and failure modes seen in electromagnetic analysis, and further, we demonstrate several means to identify, characterize, and quantify these limitations. When provided with this knowledge, a user can then proceed with increased confidence that a specific electromagnetic analysis will, or possibly will not, provide an accurate answer for a specific problem.

II. INTERPOLATION FAILURE

Interpolation of electromagnetic data has seen extensive research in recent years, for example, [1]-[2]. Much of the research has been based on the use of the Padé polynomial, a ratio of two polynomials resembling the Laplace transform of a lumped circuit. These algorithms are typically iterative and can occasionally fail. Little has been published about these failure mechanisms.

One type of failure occurs with box resonances. While circuit resonances are well modeled by the Padé polynomial, box resonances generally increase the number of iterations required by the algorithm. If too many box resonances exist in the band being analyzed, then excessive analysis time is required.

To test interpolation robustness in the presence of box resonances, a microstrip resonator is analyzed inside a box (5 × 5 mm substrate, 0.5 mm thick, ε_r=10 with 5 mm air above, line width 0.6 mm, resonator length 2.8 mm, 0.1 mm gaps, 1 mm long feed lines, cell size 0.1 × 0.1 mm, lossless.). The Sonnet Adaptive Band Synthesis (ABS^{TM}) interpolation is used [3], however, the box resonance performance of any EM interpolation can be explored.

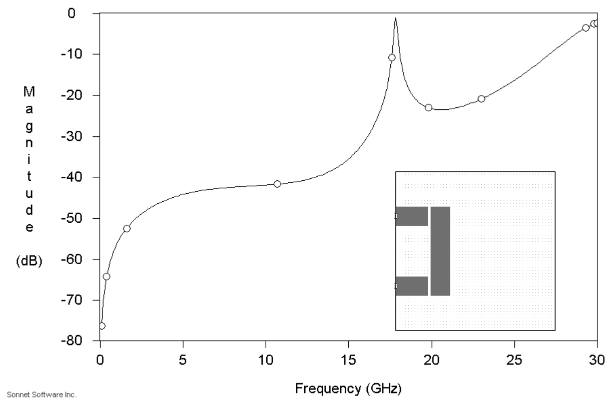

Fig. 1. Baseline analysis with circuit resonance at 17.8 GHz is handled by the interpolation with no difficulty.

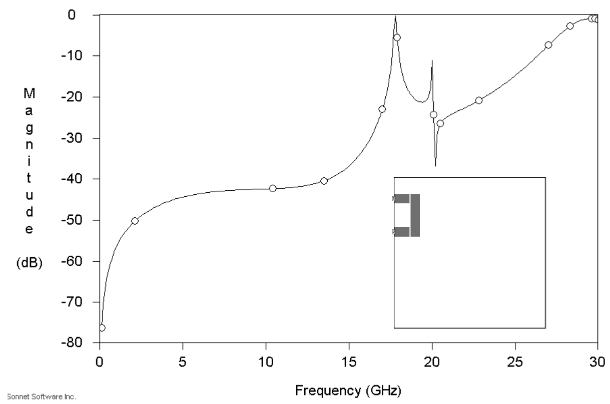

Fig. 2. Doubling the size of the box introduces a box resonance at 20.1 GHz.

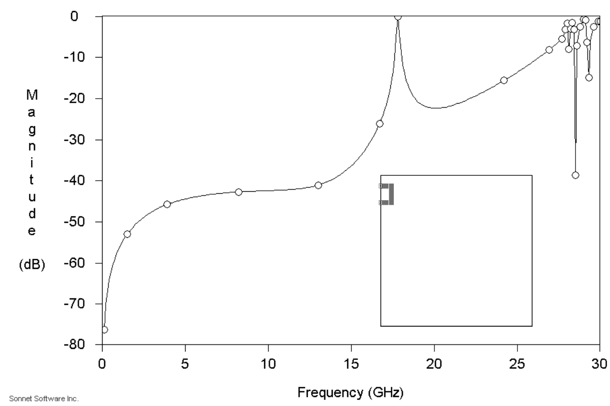

Fig. 3. Doubling the box size once more introduces a forest of resonances above 20 GHz, making interpolation difficult.

Figure 1 shows the baseline result. Analysis at 10 frequencies, indicated by data markers, is required for a full interpolation from 0.1 to 30 GHz. The 18.1 GHz circuit resonance is no problem for the interpolation However, there are resonances just above 30 GHz, already requiring a small cluster of analysis frequencies.

Figure 2 shows results for the substrate doubled to 10 × 10 mm. The single box resonance at 20.1 GHz is easily interpolated. However, the cluster of box resonances just above 30 GHz is becoming troublesome, requiring a total of 14 frequencies for the complete interpolation.

Figure 3 shows the box doubled once more, to 20 × 20 mm. Box resonances at 10.1, 15.9, and 20.0 GHz are not excited by this circuit. The forest of box resonances, now just above 25 GHz, forces the interpolation to 26 analysis frequencies, most of them clustered at the high end of the band.

From these results, we infer that this interpolation should not be used when there is a forest of box resonances, however a small number of box resonances are easily analyzed, and any box resonances not excited by a specific circuit can be ignored. Exploring the limits of interpolation using simple circuits like this can save considerable wasted effort later.

The ABS interpolation can also fail for circuits with S-parameters below –100 dB, as discussed later. Various interpolation algorithms have also been known to fail for circuits of high complexity, however, that failure mode has not been seen using this particular interpolation.

III. DYNAMIC RANGE

To stress dynamic range we analyze a simple band-stop filter. Four quarter wave stubs are attached at slightly more than half wave intervals. The circuit is in a narrow box to minimize waveguide modes, inclusion of the box is a critical part of the analysis. Using a cell size as small as possible maximizes stress on dynamic range. Dynamic range is stressed by both the small cell size (in terms of wavelengths) and by the large number of subsections (which requires a large matrix inversion). In addition, the stop band is extremely narrow, maximizing sensitivity to numerical error, i.e., S-parameters near 0 dB are very close in frequency to S-parameters at –100 dB.

All transmission lines are 0.1 mm wide, the resonators are 10 mm long, spaced every 10.6 mm and attached alternately on one side and then the other. The substrate is 91.8 × 0.7 mm, 0.1 mm thick, ε_r=10, and has 0.1 mm of air above. All analyses use a square cell size. The inset in Figure 4 shows one resonator, the entire filter is not shown due to the extreme aspect ratio (it is very long).

Unless otherwise stated, the cell size is 0.0125 mm, allowing exactly eight cells across the width of all lines. This corresponds to 3200 cells per wavelength. The cells are gradually merged into larger subsections on the interior of lines (Fig. 4), while still keeping subsections one cell wide on the line edges. The largest allowed subsection is, by default, $1/20^{th}$ of a wavelength.

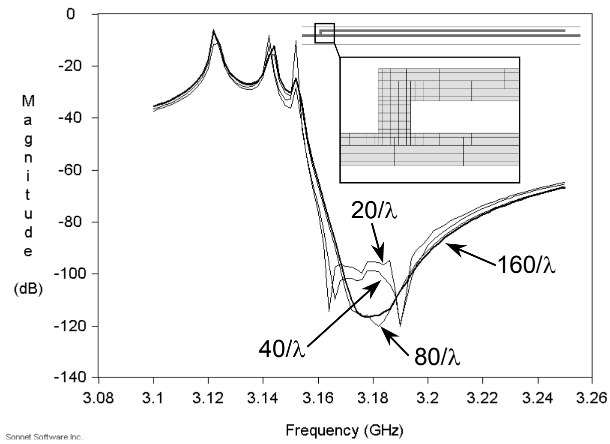

Fig. 4. Convergence analysis with respect to largest subsection size shows substantial sensitivity below –100 dB.

However, this is insufficient for this filter when accurate data is required in the band-stop region. Figure 4 shows that something on the order of 80 subsections/λ (Analysis→Advanced Subsectioning) is required for convergence to within several dB in the band-stop region of this filter. We use 160 subsections/λ (subsectioning shown in the inset of Fig. 4) for all remaining analyses.

Figure 5 shows convergence for cell size, in terms of cells per line width. Our baseline analysis assumes 8 cells per line width; 4 and 16 cells per line width are also plotted. Results below –100 dB are especially sensitive to cell size. Note that even 4 cells/width is already extremely small in terms of wavelength. That an even smaller cell size is required demonstrates that cell size must also be small with respect to line width in order to achieve accuracy [4]-[5].

Figure 6 shows the effect of dynamic range on the ABS interpolation. Analysis at four frequencies (circle data markers) is required for the complete interpolation.. No analysis frequencies fall within the stop-band, thus maximizing interpolation error. The square data markers show the frequency-by-frequency analysis. Results are visually identical except below –100 dB where the maximum difference is 3 dB. If higher accuracy is required below –100 dB, then interpolation should not be used.

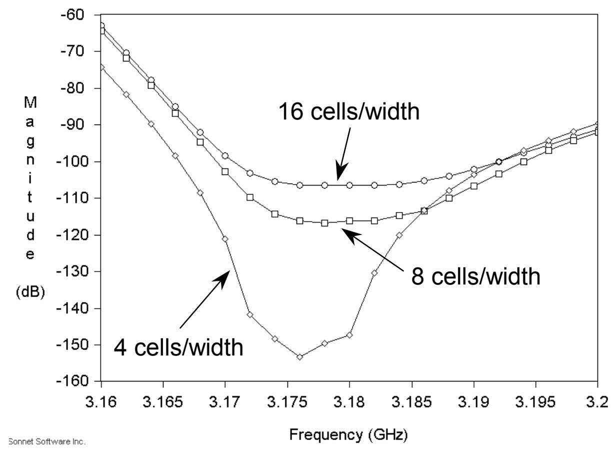

Fig. 5. Convergence analysis for cell size also shows substantial sensitivity below –100 dB (note enlarged scale).

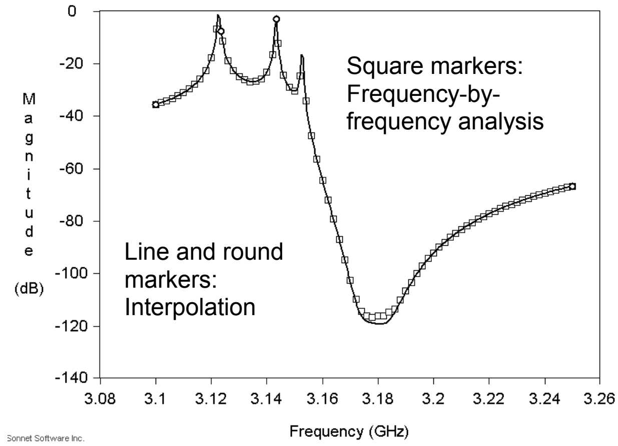

Fig. 6. The ABS interpolation requires analysis at only four frequencies, but is up to 3 dB in error below –100 dB.

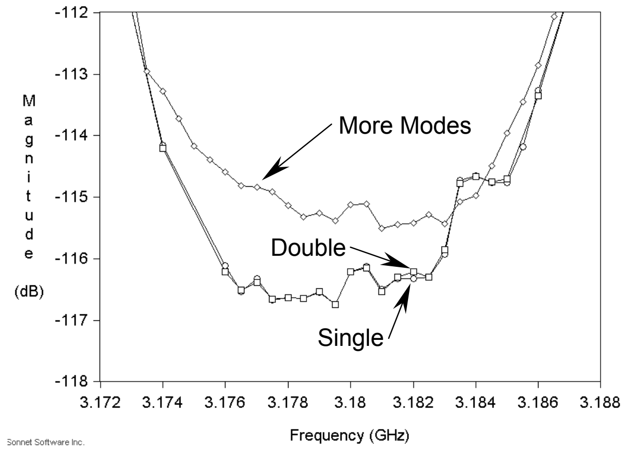

Fig. 7. Single and double precision have almost the same numerical noise, adding more modes reduces the noise.

Data in the band-stop region of Fig. 6 appears smooth. Enlarging that region significantly and quadrupling the number of data points shows there is actually about 1 dB of noisy ripple, Fig. 7. To determine how much of this noise is due to matrix solve numerical precision, the analysis was repeated in single precision, also shown in Fig. 7. The single precision result is almost identical to the double precision result, including the noise. Differences are on the order of 0.1 dB at –116.3 dB down. This corresponds to a +/– 1.77×10^{-8} difference in magnitude, suggesting a matrix solve noise floor of about –155 dB. Notice that it is important to take a large number of data points in the stop-band as most data points show almost no difference between single and double precision. A limited sample size would generate an optimistic estimate of the noise floor.

Since solving the matrix in single or double precision makes almost no difference in the result, most of the numerical noise must come from elsewhere. The third curve shows the result of approximately doubling the number of modes used by the FFT to calculate coupling between subsections (Analysis→Advanced→"-c3"). The average value of stop-band insertion loss has decreased by about one dB and the noise is about cut in half. Assuming the original peak numerical noise magnitude is about 0.5 dB peak (1.0 dB peak-to-peak, sometimes adding in phase, sometimes out of phase), we have a difference in magnitude at –115 dB of +/– 9.95×10^{-7} suggesting a noise floor of –120 dB. Keep in mind that this is an extreme case with likely contributing factors including the extremely small cell size (in terms of wavelength), and the extreme aspect ratio of the shielding box (it is very narrow).

As a rule-of-thumb we consider EM results to be reliable down to about 20 dB above the noise floor, or about –100 dB in this case. In practice, we find noise floors can range from –100 dB to –180 dB depending on the circuit being analyzed. Referring back to Fig. 5, note that in this case, error introduced by cell size overwhelms the numerical precision error of Fig. 7. Even so, we are still able to quantify the numerical precision error and even identify probable sources.

III. Conformal Mesh

FFT based planar EM analyses [6] require a fine underlying mesh in order to use the FFT and thus derive the accuracy and dynamic range benefits of the FFT. To reduce analysis time, the small FFT cells are usually merged together to form larger rectangular subsections reducing the size of the matrix to be inverted (inset, Fig. 4). Although difficult, it is also possible to merge the FFT cells into subsections that follow curving edges [7].

To test conformal meshing, all the lines in the band-stop filter of the previous section are curved into semicircles with inside radii of either 1.0 or 0.8 mm as appropriate, Fig. 8. Only one resonator of the filter is shown due to the extreme length of the substrate. The inset shows the staircasing due to the fine underlying FFT cells. Conformal meshing merges these cells together to form curving subsections, while at the same

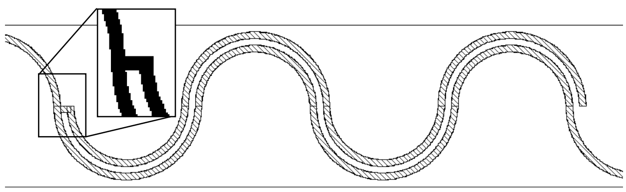

Fig 8. To test conformal meshing, the band-stop filter is meandered, one of four resonators shown.

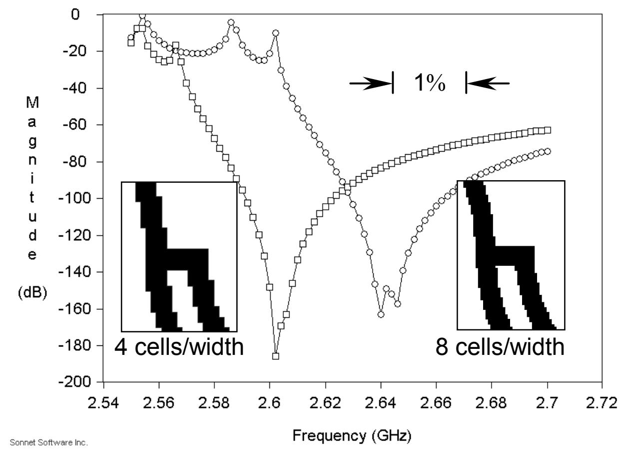

Fig. 9. Conformal mesh shows a 1.5% shift in center frequency as cell size changes, see text.

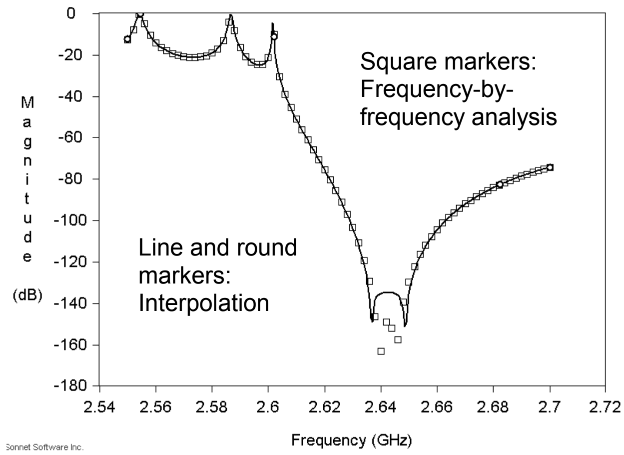

Fig. 10. Interpolation as applied to this circuit yields good results down to –120 dB.

time including the high edge current that is required for accuracy.

Figure 9 shows analysis for both 4 and 8 cells across the line width. In contrast to the straight band-stop filter results, Fig. 5, the center frequency shows a shift of 1.5%. This appears to be due to the staircase edge (insets in Fig. 9). The larger 4 cells/line width staircase slows wave propagation on the curved line. Several frequencies were analyzed at 16 cells/line width indicating a resonant frequency 0.5% higher. Thus, one percent bandwidth class curved resonator filter analysis should be performed with this potential problem in mind.

Figure 10 shows the ABS interpolation applied to the meandered and conformally meshed band-stop filter. The interpolation requires five analysis frequencies for a complete interpolation. No analysis frequencies fall within the stop-band, thus maximizing interpolation error. Interpolated data below –120 dB starts to diverge from the frequency-by-frequency calculation.

While numerical precision error for this filter was not investigated; it appears that the noise floor is likely to be under -140 dB. The better noise floor may be due to the relaxed box size aspect ratio (the substrate is wider).

VI. Conclusion

While most electromagnetic research involves demonstrating accuracy, we have taken the opposite approach by looking for analysis error and limitations. We have found, for the specific analysis investigated, that the interpolation approach fails when there is a 'forest' of box resonances, while one or two box resonances is no problem. Additionally, we have investigated dynamic range limitations and the effect of numerical precision error on interpolation by means of a band-stop filter. We have found dynamic range typically exceeds 100 dB and the specific interpolation algorithm we investigated can diverge from the frequency-by-frequency calculation for data below –100 dB. These tests were performed on both a straight filter using regular meshing and a meandered filter using the recently introduced conformal meshing. Quantitative information on limitations and analysis error is critical for efficient use of electromagnetic analysis tools.

References

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